Quality Engineering®, 17:165–171, 2005
Copyright © Taylor & Francis Inc.
ISSN: 0898-2112 print / 1532-4222 online
DOI: 10.1081/QEN-200029001

Index C^*_{pm} in Multiple Response Optimization

C. K. Ch'ng,* S. H. Quah, and H. C. Low

School of Mathematical Sciences, Universiti Sains Malaysia, USM, Penang, Malaysia

The index C^*_{pm} used in process capability is proposed as an optimization criterion in this article. This index can be viewed as an extension of mean squared error (MSE). Thus, the index C^*_{pm} allows practitioners to find an optimal setting with the mean responses close to their respective target values, T_i, while the standard deviations (or variance) of the responses are kept small. The index C^*_{pm} as an optimization criterion is applicable when upper specification limits, target values, and lower specification limits of mean responses are available. However, it also can be applied to mean responses with only target values and lower specification limits (or upper specification limits) in specific situations. An example will be used to illustrate this proposal.

Keywords index C^*_{pm}, mean squared error (MSE), upper specification limit (*USL*), lower specification limit (*LSL*), optimization, generalized reduced gradient (GRG)

INTRODUCTION

Researchers have carried out numerous studies since the introduction of Response Surface Methodology (RSM). RSM typically involves experimental design, regression models, and optimization. The relationship between the independent variables and the responses are needed in RSM. Thus, RSM normally starts with experimental design. The central composite design, the Box-Behnken design, and others are among designs used in RSM. The use of experimental designs allows practitioners to collect data on the responses at a particular design point. Furthermore, regression models are built based on the data collected in the experimental design. Four regression techniques available in the literature are: ordinary least-squares regression (OLS), generalized least squares (GLS), multivariate regression (MVR), and seemingly unrelated equations regression (SURE). Optimization, as the final step, will be done after model building. Generally, there are two types of optimization: dual response surface optimization and multiple response optimization. Dual response surface optimization is an approach that allows practitioners to optimize the primary response subject to an appropriate constraint on the value of the secondary response. Castillo and Montgomery (1993), Copeland and Nelson (1996), Fan (2000), Kim and Lin (1998), Lin and Tu (1995), Vining and Bohn (1998), Vining and Myers (1990), and others have contributed significantly to the development of dual response surface optimization. The mean response and the standard deviation (or variance) response of a quality characteristic are needed in dual response surface optimization. Table 1 is the general form of an experimental design for a dual response surface approach with m design points, k coded process settings, and n replications at each design point. Figure 1 shows a summary of the dual response surface approach.

In multiple response optimization, researchers seek to optimize the mean response of p quality characteristics simultaneously to find an optimal setting. However, the standard deviation (or variance)

*Address correspondence to C. K. Ch'ng, School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia. E-mail: chngchuankim@yahoo.com

ORDER REPRINTS

Table 1
General form of an experimental design for a dual response surface approach

	Coded process settings				Replication			
Design point	x_1	x_2	$\cdots$	x_k	1	2	$\cdots$	n
1	−1	−1	$\cdots$	−1	Y_{11}	Y_{12}	$\cdots$	Y_{1n}
2	0	−1	$\cdots$	−1	Y_{21}	Y_{22}	$\cdots$	Y_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	1	1	$\cdots$	0	Y_{m1}	Y_{m1}	$\cdots$	Y_{mn}

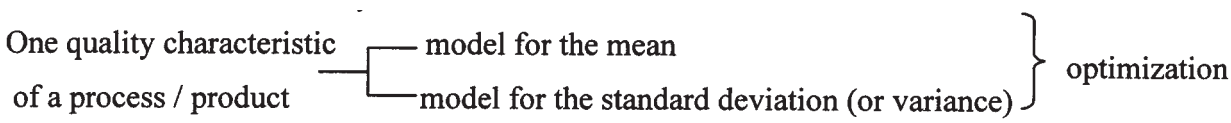

Figure 1. Summary of dual response surface approach.

of the responses of the p quality characteristics are not taken into consideration during the optimization. Castillo et al. (1996), Derringer and Suich (1980), Leon (1997), and others contributed to this kind of optimization. Table 2 is the general form of an experimental design for a multiple response approach with p quality characteristics, m design points, k coded process settings, and single observation (nonreplication) at each design point. Figure 2 shows a summary of the multiple response approach.

Besides these two approaches, researchers like Ribeiro et al. (2000–2001) combined the dual response surface approach and multiple response approach together. Such a combination allows practitioners to take the standard deviation (or variance) of the responses of the p quality characteristics into consideration during optimization. Table 3 is the general form of the combination between the dual response surface approach and the multiple response approach with p quality characteristics, m design points, k coded process settings, and n replications at each design point of every quality characteristic. Figure 3 shows the combination between the two approaches and it is our focus in this article.

In the next section, we introduce the index C^*_{pm}, which can be used when the upper specification limit (USL), the target value, and the lower specification limit (LSL), of the mean response are available. In addition, the use of the index C^*_{pm} in mean responses with only target value and LSL (or USL) will be discussed. An example will be used to illustrate the proposed optimization index.

THE INDEX C^*_{pm}

Lin and Tu (1995) proposed to minimize the mean squared error (MSE) as an optimization criterion in dual response surface and defined MSE as:

$$\mathrm{MSE} = (\hat{\omega}_\mu - T)^2 + \hat{\omega}_\sigma^2 \qquad [1]$$

where $\hat{\omega}_\mu$ is the fitted response surface for the mean; $\hat{\omega}_\sigma$ is the fitted response surface for the standard deviation, and T is the target value for the mean.

In this article, the index C^*_{pm}, which was originally proposed by Chan et al. (1988) is used as an optimization criterion. Chan et al. (1988) defined the index C^*_{pm} as:

$$C^*_{pm} = \frac{\min(USL - TT - LSL)}{3\sqrt{(\mu - T)^2 + \sigma^2}} \qquad [2]$$

$$= \frac{d - |m - T|}{3\sqrt{(\mu - T)^2 + \sigma^2}} \qquad [3]$$

where USL is the upper specification limit,
LSL is the lower specification limit,
T is the target value for the mean,
μ is the mean, σ is the standard deviation,
$d = (USL - LSL)/2$, and
$m = (USL + LSL)/2$.

The idea of introducing the index C^*_{pm} by Chan et al. (1988) is to assess the probability of falling outside the specification limits when a process with target value T is normally distributed. The original

Copyright © Marcel Dekker, Inc. All rights reserved.

Marcel Dekker, Inc.
270 Madison Avenue, New York, New York 10016

ORDER REPRINTS

Table 2
General form of an experimental design for a multiple response approach

Design point	Coded process settings				Quality characteristics			
	x_1	x_2	$\cdots$	x_k	1	2	$\cdots$	p
1	-1	-1	$\cdots$	-1	Y_{11}	Y_{12}	$\cdots$	Y_{p1}
2	0	-1	$\cdots$	-1	Y_{21}	Y_{22}	$\cdots$	Y_{p2}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	1	1	$\cdots$	0	Y_{1m}	Y_{2m}	$\cdots$	Y_{pm}

Table 3
General form of an experimental design for the combination between dual response surface approach and multiple response approach

	Coded process settings				Quality characteristic 1 replication				Quality characteristic 2 replication				Quality characteristic 3 replication			
Design point	x_1	x_2	$\cdots$	x_k	1	2	$\cdots$	n	1	2	$\cdots$	n	1	2	$\cdots$	n
1	-1	-1	$\cdots$	-1	Y_{111}	Y_{112}	$\cdots$	Y_{11n}	Y_{211}	Y_{212}	$\cdots$	Y_{21n}	Y_{p11}	Y_{p12}	$\cdots$	Y_{p1n}
2	0	-1	$\cdots$	-1	Y_{121}	Y_{122}	$\cdots$	Y_{12n}	Y_{221}	Y_{222}	$\cdots$	Y_{22n}	Y_{p21}	Y_{p22}	$\cdots$	Y_{p2n}
$\vdots$		$\vdots$				$\vdots$		$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$		$\vdots$
m	1	1	$\cdots$	0	Y_{1m1}	Y_{1m2}	$\cdots$	Y_{1mn}	Y_{2m1}	Y_{2m2}	$\cdots$	Y_{2mn}	Y_{pm1}	Y_{pm2}	$\cdots$	Y_{pmn}

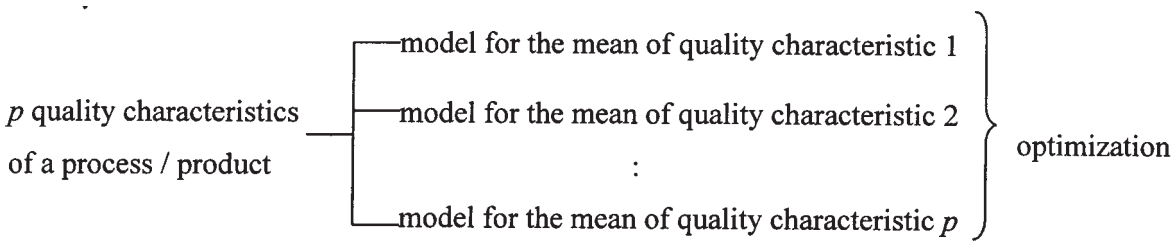

Figure 2. Summary of multiple response approach.

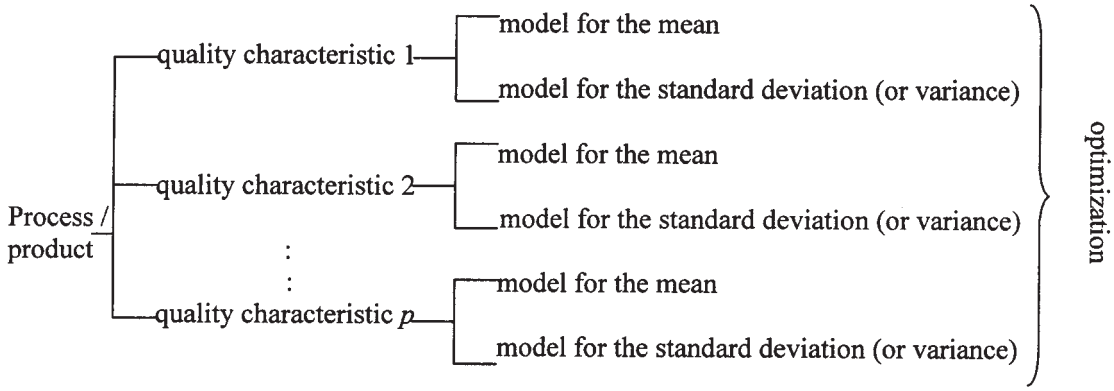

Figure 3. Summary of combination between dual response surface approach and multiple response approach.

Copyright © Marcel Dekker, Inc. All rights reserved.

Marcel Dekker, Inc.
270 Madison Avenue, New York, New York 10016

ORDER REPRINTS

idea of using the index C^*_{pm} (that is, the probability of falling outside the specification limits) will not be applied here. The index C^*_{pm} is simply used as an optimization metric in this article, and [2] and [3] are modified so that the index C^*_{pm} becomes:

$$C^*_{pm} = \frac{\min(USL - TT - LSL)}{3\sqrt{(\hat{\omega}_\mu - T)^2 + \hat{\omega}^2_\sigma}} \quad [4]$$

$$= \frac{d - |m - T\}}{3\sqrt{(\hat{\omega}_\mu - T)^2 + \hat{\omega}^2_\sigma}} \quad [5]$$

Equations [4] and [5] can be viewed as an extension of [1]. The greater the value of the index C^*_{pm}, the better the optimal settings. In order to handle a process or product with p quality characteristics, we further modify Eqs. [4] and [5] and define Total C^*_{pm} as:

$$\text{Total } C^*_{pm} = \sum_{i=1}^{p} e_i \frac{\min(USL_i - T_i T_i - LSL_i)}{3\sqrt{\hat{\omega}_{i\mu} - T_i)^2 + \hat{\omega}^2_{i\sigma}}} \quad [6]$$

$$= \sum_{i=1}^{p} e_i \frac{d_i - |m_i - T_i|}{3\sqrt{(\hat{\omega}_{i\mu} - T_i)^2 + \hat{\omega}^2_{i\sigma}}} \quad [7]$$

where USL_i is the USL quality characteristic i,
LSL_i is the LSL quality characteristic i,
T_i is the target value for the mean of quality characteristic i,
$\hat{\omega}_{i\mu}$ is the fitted response surface for the mean of quality characteristic i,
$\hat{\omega}_{i\sigma}$ is the fitted response surface for the standard deviation of quality characteristic i,
$d_i = (USL_i - LSL_i)/2$,
$m_i = (USL_i + LSL_i)/2$,
e_i is the degree of importance or priority of quality characteristic i, and
$\sum_{i=1}^{p} e_i = 1$.

When USL, target value and LSL are available, both Eqs. [6] and [7] can be used to handle "target value is best". In addition, Eq. [6] also permits the optimization of one-sided specification limits, that is, when only the target value and LSL (or USL) are available. For a quality characteristic with only the target value and LSL, following Chan et al. (1988), we let $USL = \infty$. Thus, Eqs. [4] and [6] have the following form:

$$C^*_{pm} = \frac{T - LSL}{3\sqrt{(\hat{\omega}_\mu - T)^2 + \hat{\omega}^2_\sigma}} \quad [8]$$

$$\text{Total } C^*_{pm} = \sum_{i=1}^{p} e_i \frac{T_i - LSL_i}{3\sqrt{(\hat{\omega}_{i\mu} - T_i)^2 + \hat{\omega}^2_{i\sigma}}} \quad [9]$$

In addition, Chan et al. (1988) also set $LSL = -\infty$ when only the target value and USL are available. Thus, [4] and [6] become:

$$C^*_{pm} = \frac{USL - T}{3\sqrt{(\hat{\omega}_\mu - T)^2 + \hat{\omega}^2_\sigma}} \quad [10]$$

$$\text{Total } C^*_{pm} = \sum_{i=1}^{p} e_i \frac{USL_i - T_i}{3\sqrt{(\hat{\omega}_{i\mu} - T_i)^2 + \hat{\omega}^2_{i\sigma}}} \quad [11]$$

In this article, the Generalized Reduced Gradient (GRG) algorithm is used to find the optimal setting that maximizes Eqs. [6], [7], [9], or [11]. As an alternative, optimization can also be done without the constant "3" in the denominators of Eqs. [6], [7], [9], and [11]. We cannot get an optimal setting without the numerators of Eqs. [6], [7], [9] and [11] if the p quality characteristics have different units (such as temperature, pressure and time). Thus, the main function of the numerators of Eqs. [6], [7], [9], and [11] is to standardize the p quality characteristics. After standardization, the p quality characteristics become dimensionless and they can be added to form the optimization criterion. However, the numerators of Eqs. [6], [7], [9], and [11] can be ignored when the p quality characteristics have the same unit of measure. The proposed optimization index has the following advantages: (i) degrees of importance or priority can be given to the p quality characteristics, (ii) it is dimensionless, (iii) it involves the standard deviation of the responses of p quality characteristics in the overall optimization, and (iv) Equations [6] can be used to optimize mean responses with two-sided specification limits and one-sided specification limits at the same time.

The use of index C^*_{pm} generally works well when the target value and LSL (or USL or both specification limits) are available. However, there is a disadvantage with our proposed optimization index, that is, the index C^*_{pm} is intended for quality characteristics that have a specific target value. If the target value is not available, index C^*_{pm} cannot be applied even though the LSL or USL or both specification limits are available.

Copyright © Marcel Dekker, Inc. All rights reserved.

Marcel Dekker, Inc.
270 Madison Avenue, New York, New York 10016

ORDER REPRINTS

This is a limitation of our proposal. Also, using C^*_{pm} for optimization may have negative consequences for other capability metrics such as NC (fraction nonconforming), NS (net sensitivity), and C_{pk} (see Flaig (2002)).

A review of the literature finds that Pignatiello (1993) proposed a discrete metric that is nearly a reciprocal of ours. However, there is a major difference between them in that ours is a continuous metric. That is, regression models for the mean and standard deviation (or variance) of p quality characteristics are built. These regression models are used to then find the optimum settings. Model building [e.g., mean responses or standard deviation (or variance) responses] is a common practice in RSM. Myers and Montgomery (2002) state that in the practical application of response surface methodology (RSM) it is necessary to develop an approximating model for the true response surface. Since the proposed optimization index is used to solve the multiple responses situation in RSM, it has followed the above statement given by Myers and Montgomery (2002). The Pignatiello metric (1993) does not build regression models for the mean or standard deviation of the p quality characteristics. Thus, our optimization metric is different from the Pignatiello metric, and we feel that our approach offers several potential advantages. For example, many practitioners (e.g., Quality Engineers and Process Engineers) have used process capability indices in their daily tasks for evaluating the performance of a process or product. Thus, practitioners are very familiar with the formulae, ideas, and concepts of process capability indices. Therefore, the formula for index C^*_{pm} may be readily understood and accepted by them in their effort to find optimum process settings.

EXAMPLE

Initially, we planned to use the experimental data from Fogliatto and Albin (2000) for illustrative purposes. This is because the experimental design in Fogliatto and Albin have a similar form to that of Table 3 of this article. Unfortunately, the experimental design does not allow for the building of regression models for the standard deviation of the three quality characteristics. Thus, we have used the regression models from Ribeiro et al. (2000–2001) to illustrate our proposal. In Ribeiro et al. (2000–2001) there are twenty regression models (five mean responses with *USL* and *LSL*, two mean responses with only *USL*, three mean responses with only *LSL*, and their corresponding variance models). We only use six out of the twenty regression models available, that is, $\hat{Y}_4\hat{Y}_7\hat{Y}_{10}$ and the corresponding variance models. $\hat{Y}_4\hat{Y}_7$, and $\hat{Y}_{10}$ are selected based on their having the highest R-squared values from their respective group (that is, mean responses with *USL* and *LSL*, mean responses with only *USL*, and mean responses with only *LSL*). The purpose of choosing $\hat{Y}_4\hat{Y}_7$, and $\hat{Y}_{10}$ from each group is to illustrate that our proposal can simultaneously optimize two-sided specification limits and one-sided specification limits. Table 4 gives the related information on $\hat{Y}_4\hat{Y}_7$, and $\hat{Y}_{10}$ from Ribeiro et al. (2000–2001).

Table 4
Responses and related information

Response variable	*LSL*	Target value, *T*	*USL*
$\hat{Y}_4$	21.02	30	32.98
$\hat{Y}_7$	—	65	78
$\hat{Y}_{10}$	496.42	530	—

Models $\hat{Y}_4\hat{Y}_7\hat{Y}_{10}$, and the corresponding variance models involve five coded control factors, that is, $X_k - 1 \le X_k \le 1$ $k = 1, 2, 3, 4$, and 5. Let $\hat{\omega}_{1\mu}$ denote the mean response of $\hat{Y}_4\hat{\omega}_{2\mu}$ denote the mean response of $\hat{Y}_7\hat{\omega}_{3\mu}$ denote the mean response of $\hat{Y}_{10}\,\hat{\omega}^2_{1\sigma}$ denote the variance of $\hat{Y}_4\,\hat{\omega}^2_{2\sigma}$ denote the variance response of $\hat{Y}_7$, and $\hat{\omega}^2_{3\sigma}$ denote the variance response of $\hat{Y}_{10}$. Thus, the fitted regression models for $\hat{\omega}_{1\mu}\,\hat{\omega}^2_{1\sigma}\,\hat{\omega}_{2\mu}\,\hat{\omega}^2_{2\sigma}\,\hat{\omega}_{3\mu}$, and $\hat{\omega}^2_{3\sigma}$ are respectively:

$$\hat{\omega}_{1\mu} = 31.57 + 3.60X_1 + 1.43X_1^2 + 1.98X_2 + 1.58X_2^2 + 1.69X_3 + 1.10X_4 + 2.36X_5 \quad [12]$$

$$\hat{\omega}^2_{1\sigma} = 0.623 + 0.253X_2 \quad [13]$$

$$\hat{\omega}_{2\mu} = 74.11 - 1.17X_1 - 4.88X_4 + 1.47X_5 + 0.92X_1X_2 - 0.689X_3X_4 \quad [14]$$

$$\hat{\omega}^2_{2\sigma} = 0.5 \quad [15]$$

$$\hat{\omega}_{3\mu} = 520.7 - 58.1X_1 - 32X_1^2 - 34.2X_2 - 22.6X_2^2 - 32.7X_3 - 12.1X_4 - 21.6X_5 \quad [16]$$

$$\hat{\omega}^2_{3\sigma} = 13.329 - 6.566X_2 - 6.673X_3 \quad [17]$$

The GRG algorithm in the "Solver" option in Microsoft® Excel® is used to maximize Eq. [6]

Copyright © Marcel Dekker, Inc. All rights reserved.

Marcel Dekker, Inc.
270 Madison Avenue, New York, New York 10016

ORDER REPRINTS

to find the optimal setting. We assume that the three quality characteristics are equally important in this article, that is, $e_1 = e_2 = e_3 = 1/3$. In addition, we also assume that the LSL of $\hat{Y}_7$ is equal to $-\infty$ and the USL of $\hat{Y}_{10}$ is equal to ∞. Furthermore, the starting point of this example is $X' = (0, 0, 0, 0, 0)$. Table 5 summarizes the optimal setting, expected mean, expected variance, index C^*_{pm}, and Total C^*_{pm}.

The maximization of Eq. [6] leads to $\hat{\omega}_{1\mu} = 28.947$ $\hat{\omega}^2_{1\sigma} = 0.715$ $\hat{\omega}_{2\mu} = 78.000$ $\hat{\omega}_{2\sigma} = 0.500$, $\hat{\omega}_{3\mu} = 529.914$, and $\hat{\omega}^2_{3\sigma} = 4.262$. This optimal setting gives the expected means of $\hat{Y}_{10}(= 529.914)$ and $\hat{Y}_4(= 28.947)$ which are very close to their respective target values and with very small expected variances. However, the expected mean of $\hat{Y}_7 = 78.000$ is far from its target value $(= 65)$ with constant expected variance. Although the proposal of this article may be viewed as an extension of MSE, it will not promise that all expected means would have only small biases from their respective target values because the proposed optimization index involves p quality characteristics in the optimization. Figure 4 gives the Microsoft® Excel® after spreadsheet implementation used for this example.

Table 5
Optimum results

Expected mean	Expected variance	C^*_{pm}	
$\hat{\omega}_{i\mu} = 28.947$	$\hat{\omega}^2_{1\sigma} = 0.715$	0.245	Total
$\hat{\omega}_{2\mu} = 78.000$	$\hat{\omega}^2_{2\sigma} = 0.500$	0.111	$C^*_{pm} = 2.162$
$\hat{\omega}_{3\mu} = 529.914$	$\hat{\omega}^2_{3\sigma} = 4.262$	1.806	

Optimal setting (−0.794, 0.365, 1.000, −0.843, − 1.000).

Based on the Ribeiro et al. (2000–2001) method, the optimal setting is given as (− 0.645, 0.475, 0.955, 1.000, −1.000). This setting yields $\hat{\omega}_{1\mu} = 31.494$ $\hat{\omega}^2_{1\sigma} = 0.743$ with $C^*_{pm} = 0.192$; $\hat{\omega}_{2\mu} = 67.575$ $\hat{\omega}^2_{2\sigma} = 0.500$ with $C^*_{pm} = 0.541$; and $\hat{\omega}_{3\mu} = 501.539$ $\hat{\omega}^2_{3\sigma} = 3.837$ with $C^*_{pm} = 0.131$. The C^*_{pm} of $\hat{Y}_7$ at this setting obviously shows some improvement over the proposed optimization index but we obtain less

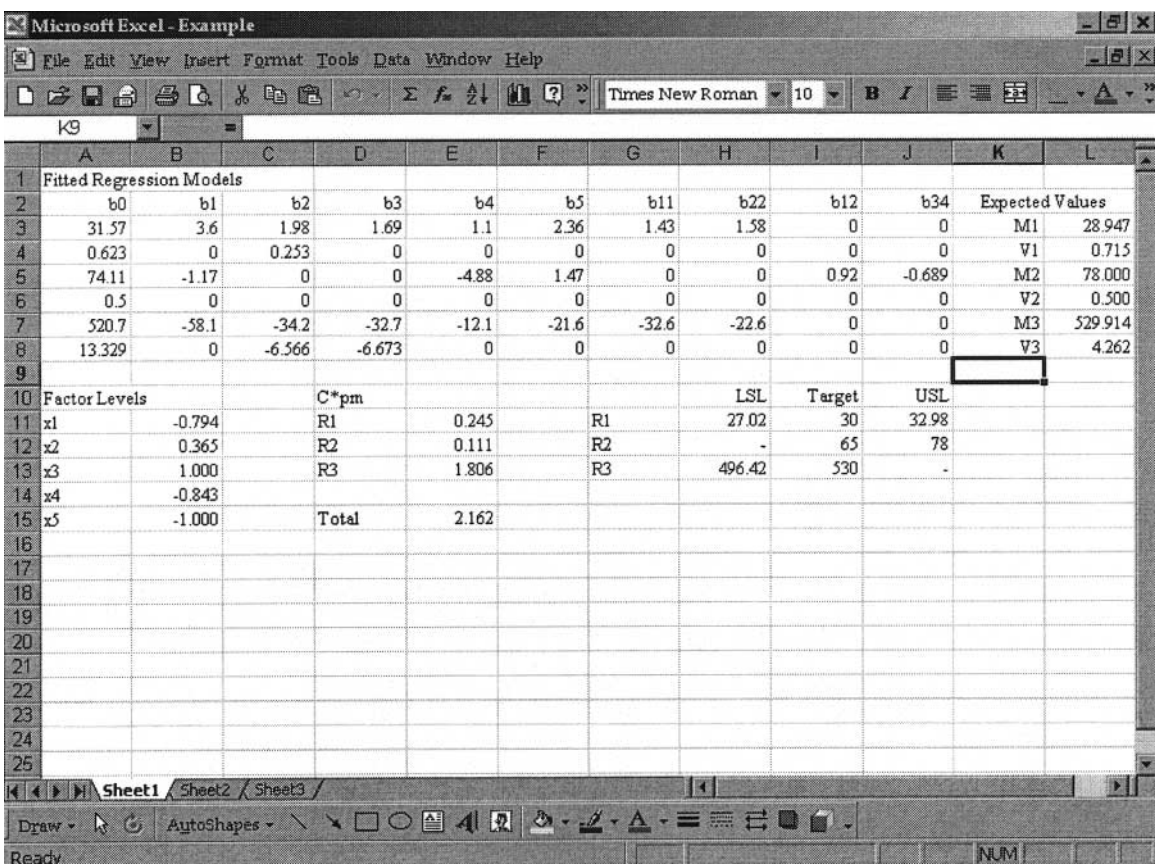

Figure 4. Microsoft® Excel® implementation.

Copyright © Marcel Dekker, Inc. All rights reserved.

Marcel Dekker, Inc.
270 Madison Avenue, New York, New York 10016

ORDER REPRINTS

favorable C^*_{pm}'s for the other two responses, especially $\hat{Y}_{10}$. The Total C^*_{pm} for this setting equals 0.864, which is smaller than that in Table 5. Thus, the proposed optimization index gives a better optimal setting for Total C^*_{pm} than the Ribeiro et al. (2000–2001) method.

CONCLUSION

The use of Total C^*_{pm} as an optimization criterion, as illustrated in this article, allows practitioners to consider both means and standard deviations (or variance) responses in the optimization. Assuming a process target value, this proposal can not only be used to handle two-sided specification limits (*USL* and *LSL*) but it can also be applied to mean response with only a *LSL* or *USL*. We believe that the use of response surface methods applied to the mean and standard deviations responses offers significant advantages in optimization. We hope that, in the future, more multiresponse experiments will be conducted along the line suggested in this article.

ABOUT THE AUTHORS

C. K. Ch'ng is a doctoral student in Statistics at the School of Mathematical Sciences, University of Science of Malaysia. His research interests are statistical process control and response surface methodology.

S. H. Quah is a Professor of Statistics in the School of Mathematical Sciences, University of Science of Malaysia. His fields of interest are quality management, statistical process control, and statistical design of experiments.

H. C. Low is an Associate Professor of Statistics in the School of Mathematical Sciences, University of Science of Malaysia. Her fields of interest are statistical process control, medical statistics, and signal processing.

REFERENCES

Castillo, E. D., Montgomery, D. C. (1993). A nonlinear programming solution to the dual response problem. *Journal of Quality Technology* 25(3):199–204.

Castillo, E. D., Montgomery, D. C., McCarville, D. R. (1996). Modified desirability functions for multiple response optimization. *Journal of Quality Technology* 28(3):337–345.

Chan, L. K., Cheng, S. W., Spiring, F. A. (1988). A new measure of process capability: C_{pm}. *Journal of Quality Technology* 20(3):162–175.

Copeland, K. A. F., Nelson, P. R. (1996). Dual response optimization via direct function minimization. *Journal of Quality Technology* 28(3):331–336.

Derringer, G., Suich, R. (1980). Simultaneous optimization of several response variables. *Journal of Quality Technology* 12(4):214–219.

Fan, S. K. S. (2000). A generalized global optimization algorithm for dual response systems. *Journal of Quality Technology* 32(4):444–456.

Flaig, J. J. (2002). Process capability optimization. *Quality Engineering* 15(2):233–242.

Fogliatto, F. S., Albin, S. L. (2000). Variance of predicted response as an optimization criterion in multiresponse experiments. *Quality Engineering* 12(4):523–533.

Kim, K. J., Lin, D. K. J. (1998). Dual response surface optimization: a fuzzy modeling approach. *Journal of Quality Technology* 30(1):1–10.

Leon, N. A. (1996–1997). A pragmatic approach to multiple-response problems using loss functions. *Quality Engineering* 9(2):213–220.

Lin, D. K. J., Tu, W. (1995). Dual response surface optimization. *Journal of Quality Technology* 27(1):34–39.

Myers, R. H., Montgomery, D. C. (2002). *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*. 2nd ed. New York: John Wiley and Sons, Inc.

Pignatiello, J. J. Jr. (1993). Strategies for robust multiresponse quality engineering. *IIE Transaction* 25:5–15.

Ribeiro, J. L. D., Fogliatto, F. S., Caten, C. S. (2000–2001). Minimizing manufacturing and quality costs in multiresponse optimization. *Quality Engineering* 13(2):191–201.

Vining, G. G., Bohn, L. L. (1998). Response surface for the mean and variance using a nonparametric approach. *Journal of Quality Technology* 30(3):282–291.

Vining, G. G., Myers, R. H. (1990). Combining taguchi and response surface philosophies: a dual response approach. *Journal of Quality Technology* 22(1):38–45.

Copyright © Marcel Dekker, Inc. All rights reserved.

Marcel Dekker, Inc.
270 Madison Avenue, New York, New York 10016

Request Permission or Order Reprints Instantly!

Interested in copying and sharing this article? In most cases, U.S. Copyright Law requires that you get permission from the article's rightsholder before using copyrighted content.

All information and materials found in this article, including but not limited to text, trademarks, patents, logos, graphics and images (the "Materials"), are the copyrighted works and other forms of intellectual property of Marcel Dekker, Inc., or its licensors. All rights not expressly granted are reserved.

Get permission to lawfully reproduce and distribute the Materials or order reprints quickly and painlessly. Simply click on the "Request Permission/ Order Reprints" link below and follow the instructions. Visit the U.S. Copyright Office for information on Fair Use limitations of U.S. copyright law. Please refer to The Association of American Publishers' (AAP) website for guidelines on Fair Use in the Classroom.

The Materials are for your personal use only and cannot be reformatted, reposted, resold or distributed by electronic means or otherwise without permission from Marcel Dekker, Inc. Marcel Dekker, Inc. grants you the limited right to display the Materials only on your personal computer or personal wireless device, and to copy and download single copies of such Materials provided that any copyright, trademark or other notice appearing on such Materials is also retained by, displayed, copied or downloaded as part of the Materials and is not removed or obscured, and provided you do not edit, modify, alter or enhance the Materials. Please refer to our Website User Agreement for more details.

Request Permission/Order Reprints

Reprints of this article can also be ordered at
http://www.dekker.com/servlet/product/DOI/101081QEN200029001