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COMPARING METHODS FOR THE MULTI-RESPONSE DESIGN PROBLEM

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SUMMARY

One approach to solving multiple response engineering problems is to combine the individual responses into one unifying objective. In utility theory, several characteristics are used to compare and contrast multiple objective techniques. These are risk aversion, marginal rates of substitution, and the relationship of the responses in the combined function. Perhaps unknown to the user, multiple response techniques carry strong assumptions regarding these characteristics. This paper investigates four commonly-used multiple objective techniques and demonstrates that each method contains assumptions about these characteristics which are not intuitively evident to a user. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS: multiresponse; optimization; multivariate; desirability

1. INTRODUCTION

Manufactured products are typically characterized by numerous quality characteristics. This type of design typically involves choosing a set of input factors that will result in a product with the best combination of these quality characteristics. Commonly this process is referred to as multi-response design optimization with the quality characteristics defined as the multiple responses and denoted by Y_i. The objective is to define a set of factors that provides the best compromise of the multiple responses simultaneously. The use of multiple objective decision making (MODM) techniques may be used to solve multi-response engineering design problems.

A common approach to solving multi-response design problems is a unifying objectives approach; that is, the individual responses are mathematically combined to form a single function. Unifying objective approaches are used in the quality area to simultaneously optimize several responses. Initially, the individual responses are modeled by creating a response surface from an experimental design. The set of response surfaces is then subjected to a mathematical transformation which acts as a normalizing agent so all responses can then be combined into a single function. Ultimately, by varying the levels of the input factors, an optimal objective function and hence optimal input factor settings can be obtained.

The optimization of a single response function is straightforward using calculus-based methods, such as gradient-based algorithms and Hooke and Jeeves pattern searches. While there are several multi-response techniques available, little has been said regarding the implicit assumptions underlying these techniques. The assumptions studied here are response relationship in the combined function, marginal rates of substitution, and user risk aversion; these are defined in subsequent sections. These three characteristics may be used to compare and contrast multi-response optimization techniques. This paper illustrates the effect of the implicit assumptions of these characteristics on the optimal response of four multi-response methods. In other words, different optimal factor settings may result from the combination of different multi-response methods and different underlying assumptions. Multiple response techniques are also investigated by Khuri and Conlon [1], Vining [2], and Wurl and Albin [3] but not with respect to the aforementioned characteristics.

The four multi-response optimization methods studied are (i) the desirability method [4], (ii) the

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Table 1. Response parameters for the multiple response example

Response—description	Min.	Max.	Target
Y_1 = maximum temperature at position A	185	195	190
Y_2 = beginning bond temperature at position A	170	195	185
Y_3 = finish bond temperature at position A	170	195	185
Y_4 = maximum temperature at position B	185	195	190
Y_5 = beginning bond temperature at position B	170	195	185
Y_6 = finish bond temperature at position B	170	195	185

modified desirability method [5], (iii) the Global Criterion method [6], and (iv) the distance approach proposed by Khuri and Conlon [1].

An example of a wire bonding process is used to illustrate the effect of the implicit assumptions in each of the methods. We use the multiple response example of Del Castillo *et al.* [7] to illustrate these concepts. The input variables in the experiment are the N_2 flow rate (x_1), the N_2 temperature (x_2), and the temperature of the heater block (x_3). The goal of the experiment is to find operating conditions that achieve optimal temperatures during the bonding process while not exceeding the melting temperature of the plastic package. Six responses are observed during the experiment, and Table 1 defines the responses and the acceptable ranges. Del Castillo *et al.* [7] modify the desirability function so that it is differentiable everywhere and gradient-based optimization methods can be used.

The following sections describe the four multi-response optimization methods. In turn, the independence conditions required by each of the methods are uncovered and the marginal rates of substitution for each approach are derived and the implications discussed. Finally, it is shown that each of the four approaches carries with it a distinct risk preference structure, which in many cases is transparent to the decision-maker.

2. MULTI-RESPONSE OPTIMIZATION TECHNIQUES

2.1. *The desirability method*

The desirability method [4], shown in (1), is a multiplicative model,

$$D = (d_1 d_2 d_3 \ldots d_n)^{1/n} \tag{1}$$

See Stevens and Baker [8] and Layne [9] for use of this method in industrial examples. D is defined as the overall desirability, and d_i is defined as an individual response desirability. The method involves the conversion of responses, Y_i, to desirability functions, d_i, using a transformation. The value of d_i increases as the desirability of the corresponding response, Y_i, increases. The rationale behind using the multiplicative model is that if any one response d_i has a low value (i.e., $d_i \approx 0$) then the overall desirability D will be unacceptable regardless of the values taken on by the other response d_i's.

The d_i's take on values between 0 and 1. This scale corresponds to a completely undesirable level of quality when $d_i = 0.00$ and a completely acceptable level of quality when $d_i = 1.00$. Harrington [4] provides the following 'subjective scale' to guide a quality engineer when assessing desirability: the scale of d_i is a dimensionless scale so each term can be interpreted in terms of desirability. Two transformations lead to the development of the individual d_i's. The two-sided transformation is for the case where a target value is sought. The desirability is highest at this target value and decreases as a response moves away from the target value in either direction. The transformation from the measurement of the property to the scale of d is accomplished using an exponential equation,

$$d = \mathrm{e}^{-(|Y'|)^n} \tag{2}$$

where e is the logarithmic constant, n is a positive number, and Y' is a linear transform of the response variable. Harrington [4] uses a linear transform on Y,

$$Y_i' = \frac{2Y_i - (Y_{\max} + Y_{\min})}{Y_{\max} - Y_{\min}} \tag{3}$$

The exponent, n, determines the slope of the curve and as n becomes large the curve becomes more bell shaped.

The one-sided transformation is used when a response is to be maximized. In this case, the desirability increases as the response increases. Low values of desirability are analogous to low levels of a response. Harrington [4] chooses a special form of exponential function to model desirability: the

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Gompertz growth curve, which is represented as

$$d_i = \mathrm{e}^{-(\mathrm{e}^{-Y'})} \tag{4}$$

In this equation the slope is determined by the scaling of Y onto Y'. The scaling is accomplished by selecting two values of the response, Y, and assigning desirability levels. These desirability levels are transformed to their equivalent Y' by the equation

$$Y' = -[\ln(-\ln d)] \tag{5}$$

From these paired values of Y and Y', the linear transformation equation of the form $Y' = b_0 + b_1 Y$ is easily derived by calculating the two constants, b_0 and b_1, by substitution. The shape of the desirability shifts as the level of b_0 and b_1 change. Many researchers find the S-shaped characteristics of the curve model an individual's utility very accurately.

2.2. *The modified desirability method*

This method by Derringer and Suich [5] is also a multiplicative method and defined as

$$D = (d_1^{w_1} d_2^{w_2} d_3^{w_3} \ldots d_n^{w_n})^{1/\Sigma w_i} \tag{6}$$

where w_i is defined as a response weight. This is a weighted composite desirability, and the concepts and function of D and d_i are identical to the Desirability method.

Derringer and Suich [5] propose another type of one-sided transform given by

$$d_i = \begin{cases} 0 & Y_i \le Y_{i\text{-min}} \\ \left[\dfrac{(Y_i - Y_{i\text{-min}})}{(Y_{i\text{-max}} - Y_{i\text{-min}})}\right]^r & Y_{i\text{-min}} \le Y_i \le Y_{i-\text{max}} \\ 1 & Y_i \ge Y_{i\text{-max}} \end{cases} \tag{7}$$

where Y_i is the predicted value for response i, $Y_{i\text{-min}}$ is the minimum acceptable value for response i, $Y_{i\text{-max}}$ is the maximum acceptable value for response i, and r is a weight used to determine scale of desirability (e.g., a larger r is used if it is very desirable for Y_i to be close to $Y_{i\text{-max}}$).

For the two-sided case, the transformation is

$$d_i = \begin{cases} \left[\dfrac{(Y_i - Y_{i\text{-min}})}{(\tau_i - Y_{i\text{-min}})}\right]^s & \text{if } Y_{i\text{-min}} \le Y_i \le \tau_i \\ \left[\dfrac{(Y_i - Y_{i\text{-max}})}{(\tau_i - Y_{i\text{-min}})}\right]^t & \text{if } \tau_i \le Y_i \le Y_{i\text{-max}} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where s and t, are user-specified weights the allow the decision maker to specify the underlying form of the desirability function.

While the Derringer and Suich transformation looks similar to Harrington's presentation of the Gompertz transformation, each has assumptions regarding the underlying structure of the desirability. Namely, Derringer and Suich assume a linear function that can be shaped using a power factor; while the Gompertz transformation uses an exponential function that can be weighted.

2.3. *The Global Criterion method*

Unlike the multiplicative methods, the Global Criterion method [6] uses an additive function to combine the individual responses as given in (9),

$$F = \sum_{i=1}^{k} \left[\frac{(Y_i - Y_{i-\text{min}})}{(Y_{i\text{-max}})}\right]^p \tag{9}$$

This method uses a ratio of the sum of the deviations of the values of the individual objective functions from their respective target values to that of the target values. From the set of objective functions, a single objective function is then formulated and solved using univariate optimization techniques.

For the maximization of a response, the transformation is

$$d_i = \left[\frac{(Y_i - Y_{i\text{-min}})}{(Y_{i\text{-max}})}\right]^p \tag{10}$$

where $Y_{i\text{-max}}$ is the value of the objective function i at its maximum acceptable level, $Y_{i\text{-min}}$ is the value of the objective function i at its minimum acceptable level and Y_i is the predicted value of response i. p is a weight used to determine scale of importance. Setting $p = 1$ gives equal importance to all deviations, whereas $p = 2$ gives more weight to larger deviations.

It can be seen from (10) that $Y_{i\text{-max}}$ is a constant and can be factored out, which yields

$$d_i = \frac{1}{(Y_{i\text{-max}})^p}[(Y_i - Y_{i\text{-min}})]^p \tag{11}$$

and substituting k_i for $1/(Y_{i\text{-max}})^p$,

$$d_i = k_i[(Y_i - Y_{i\text{-min}})]^p \tag{12}$$

This representation is a simple linear function anchored at a maximum level of the response. In the case where $i = 1$ to 3 the global criterion approach resembles a weighted additive model. This can be seen

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as follows,

$$F = \left[\frac{1}{(Y_{1\text{-max}})}\right] d_1 + \left[\frac{1}{(Y_{2\text{-max}})}\right] d_2 + \left[\frac{1}{(Y_{3\text{-max}})}\right] d_3 \tag{13}$$

The term $[1/(Y_{i\text{-max}})]$ can be interpreted as the weight associated with each response.

2.4. *The Khuri and Conlon distance measure*

The Khuri and Conlon (henceforth, KC) distance measure [1] is also an additive function:

$$\hat{D}_{\mathrm{KC}}(x) = \sqrt{[\hat{Y}(x) - \tau]' \operatorname{Var}[\hat{Y}(x)]^{-1} [\hat{Y}(x) - \tau]} \tag{14}$$

where the term $\operatorname{Var}[\hat{Y}(x)]$ is a $p \times p$ matrix of the variance of the prediction models of $Y(x)$. To understand the structure of the model, one should consider the case where an experimenter has p uncorrelated response variables. Allowing $\sigma_i^2 =$ the variance of Y_i and $\widehat{\operatorname{Var}}[\hat{Y}(x)] = z'(x)(X'X)^{-1} z(x) \hat{\Sigma}$, equation (14) reduces to

$$\hat{D}_{\mathrm{KC}}(x) = \left[\frac{1}{v(x)} \sum_{i=1}^{p} \left(\frac{[\hat{Y}_i(x) - \tau_i]}{\sigma_i}\right)^2\right]^{1/2} \tag{15}$$

where $v(x) = z'(x)(X'X)^{-1} z(x)$. This model uses the squared deviations of the response characteristics from their targets, but then normalizes these deviations by the variance of the response predictions.

Khuri and Conlon [1] propose this method for determining the response targets, τ_i. For response variables where the target is smaller-is-better (larger-is-better), τ_i is set equal to the minimum (maximum) of the response over the experimental region. In the case of target-is-better, the value for τ_i is set equal to the target if there exists a setting within the experimental region that yields τ_i.

Khuri and Conlon [1] do not explicitly use a transformation as the previous methods do. Instead, the distance metric itself implicitly assigns a transform. This is seen directly in the uncorrelated case developed in (15). The implicit transform is very similar to the transform in Tabucanon [6]. It is the squared value of an underlying linear function. Allowing

$$d_i = \left(\frac{[\hat{Y}_i(x) - \tau_i]}{\sigma_i}\right)^2 \tag{16}$$

it is clearly seen that σ_i is a constant and can be factored out, therefore the same relationship as in (12) is realized.

The model accounts for the correlations between the responses by employing the variance/covariance structure of the distance metric. As we will show later, this fact also allows the variance/covariance structure to dictate the marginal rates of substitution or sensitivity between the responses.

3. RESPONSE RELATIONSHIP IN THE COMBINED OBJECTIVE FUNCTION

This section looks at the relationships between the responses that are implicit in the multi-response optimization methods. In general, multi-response optimization methods employ either a strictly additive function or strictly multiplicative function [10]. The desirability and modified desirability methods are multiplicative, while the global criterion and KC distance measure methods are additive models.

Using the data from the Del Castillo *et al.* [7] experiment, optimal sets of input factors and responses were derived for each of the four unifying methods. Table 2 displays the optimal response settings. The left-hand column displays the optimum values for the factor settings (x_1, x_2, x_3) both in coded form and uncoded form, along with the associated response settings $(Y_1, Y_2, Y_3, Y_1, Y_2, Y_3)$ for each model.

The advantage of the additive functions is that the responses are simply summed. However, this may have the effect of driving the optimal factor settings to their extreme values instead of the target values. On the other hand, the multiplicative tend toward the target, and the multiplicative functions derive a solution based on the relationship between responses.

As the optimization process takes place, the multiplicative functions tend to move away from the minimum and maximum values of the input factors. In short, if any one response is pushed to a maximum or minimum value, the corresponding individual desirability will tend towards zero resulting in a lower overall desirability.

This is noticeable when the final response values, Y_i, are examined in Table 2. It can be seen that the desirability and modified desirability methods have one optimal set of input factor settings. Conversely, two optimal combinations of factor settings exist for the Global Criterion and KC distance methods. In addition, none of the responses in the desirability methods are at their minimum or maximum values. However, in the case of the additive models there is at least one case in each solution where a response is set

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Table 2. Optimal response settings

Results	Desirability	Modified desirability	Global criterion		KC distance measure	
Coded						
x_1	0.1043	0.0636	0.2129	0.6677	0.1756	1.0000
x_2	1.0000	1.0000	1.0000	0.9132	1.0000	0.8616
x_3	0.7966	0.7785	0.8654	0.5579	0.8654	0.7901
Uncoded						
x_1	84.17	82.54	88.51	106.71	87.02	120.00
x_2	450.00	450.00	450.00	439.15	450.00	432.70
x_3	329.66	327.85	336.54	305.79	336.54	329.01
Response settings						
Y_1	186.06	186.34	**185.00**	188.62	**185.00**	**185.00**
Y_2	174.49	173.99	176.12	172.63	175.89	176.79
Y_3	172.02	171.37	174.17	**170.00**	173.89	176.17
Y_4	192.63	191.72	**195.00**	**195.00**	**194.36**	**194.88**
Y_5	173.04	172.32	175.23	171.27	174.87	174.30
Y_6	184.95	184.10	187.71	180.56	187.41	186.63

at its minimum or maximum value and/or extremely close to its minimum or maximum. These cases are highlighted in bold in Table 2.

4. MARGINAL RATES OF SUBSTITUTION

Within mathematical programming, substitution or trade-off information is referred to as dual information. In situations with convex settings, Lagrange multipliers are used to describe the trade-offs between alternatives. In economics, these trade-offs are commonly defined as marginal rates of substitution and usually given as prices. Prices, for example, are the basis for what one commodity or service is worth compared to another commodity or service (see Keeney and Raiffa [10], Bogetoft and Pruzan [11], and Nakayama and Sawaragi [12]).

In the context of a multi-response quality design problem, this translates into how much of one response a user is willing to give up in order to acquire one unit of another response or the sensitivity of the responses with regard to one another [2]. Thus, trade-off information or marginal rates of substitution are highly useful in multi-response quality design problems for they determine how a decision maker feels about one response weighed against another response.

The desirability function contains some unique information regarding the trade-offs between responses. As stated earlier, the desirability function is a multiplicative function where the scaling constants are identical and equal to $1/n$. The scaling constants in the multiplicative function represent the marginal rate of substitution for the responses. For example, the marginal rate of substitution, λ, of $u(d_1, d_2)$ is roughly the amount of d_1 one can forego for a unit of d_2. The marginal rate of substitution can be obtained from

$$\lambda_{d_i,d_j} = \frac{\partial u_{d_i}(d_1, d_2, \ldots, d_n)}{\partial u_{d_j}(d_1, d_2, \ldots, d_n)} \tag{17}$$

For the desirability function, the marginal rate of substitution for any λ of $u(d_1, d_2, \ldots, d_n)$ is as follows for $i > 1$, $j > 1$, and $i \neq j$,

$$\lambda_{d_i,d_j} = \frac{\partial u_{d_i}(d_1, d_2, \ldots, d_n)}{\partial u_{d_j}(d_1, d_2, \ldots, d_n)} = \frac{1/n}{1/n} = 1 \tag{18}$$

Therefore, the marginal rate of substitution for all attributes d_i in the desirability approach are equal. In other words, the decision maker is willing to give up an equal amount of one attribute to gain that amount in another attribute.

Completing the same marginal analysis using the modified desirability method, a different conclusion is reached. Using (17), the marginal rates of substitution for any λ of $u(d_1, d_2, \ldots, d_n)$ for $i > 1$, $j > 1$, and $i \neq j$ are unique, therefore, the marginal rates of substitution are

$$\lambda_{d_i,d_j} = \frac{\partial u_{d_i}(d_1, d_2, \ldots, d_n)}{\partial u_{d_j}(d_1, d_2, \ldots, d_n)} = \frac{w_i/\Sigma w_n}{w_j/\Sigma w_n} = \frac{w_i}{w_j} \tag{19}$$

From this general relationship, it is seen that the marginal rate of substitution between two responses is the ratio of the exponents chosen in (6).

The Global Criterion method contains implicit marginal rates of substitution based on a responses

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Table 3. Marginal rates of substitution for response pairs

Response pairs	Method				
	Desirability	Modified desirability	Global criterion	KC uncorrelated	KC correlated
1–2	1	2	3.9×10^{-53}	**0.521278**	
1–3	1	2	1.8×10^{-52}	**0.42494**	
1–4	1	1	5.4×10^{-54}		**−0.66516**
1–5	1	2	7.3×10^{-53}	**0.473527**	
1–6	1	2	5.5×10^{-55}	**−2.84276**	
2–3	1	1	**4.65131**		1.001776
2–4	1	0.5	**0.13999**	**−3.56395**	
2–5	1	1	**1.87166**		1.006732
2–6	1	1	**0.01422**		0.993577
3–4	1	0.5	**0.0301**	**−4.57037**	
3–5	1	1	**0.40239**		1.005339
3–6	1	1	**0.00306**		1.156903
4–5	1	2	**13.3699**	−0.25488	
4–6	1	2	**0.10157**	1.195392	
5–6	1	1	**0.0076**		1.166512

maximum value, minimum value, and the value of the response at a specific point. From (9), the marginal rates of substitution for any λ of $u(d_1, d_2, \ldots, d_n)$ for $i > 1$, $j > 1$, and $i \neq j$ are unique. In general, the marginal rates of substitution are

$$\lambda_{d_i,d_j} = \frac{\partial u_{d_i}(d_1, d_2, \ldots, d_n)}{\partial u_{d_j}(d_1, d_2, \ldots, d_n)} = \frac{(Y_j - Y_{j\text{-min}})[(Y_i - Y_{i\text{-min}})/(Y_{i-\text{max}})]^P}{(Y_i - Y_{i\text{-min}})[(Y_j - Y_{j\text{-min}})/(Y_{j-\text{max}})]^P} \tag{20}$$

These rates are subject to $(Y_i - Y_{i\text{-min}}) \neq 0$ and $(Y_j - Y_{j\text{-min}}) \neq 0$.

Initially, some interesting conclusions regarding (20) can be made. The two quantities, $[(Y_i - Y_{i\text{-min}})/(Y_{i\text{-max}})]^P$ and $[(Y_j - Y_{j\text{-min}})/(Y_{j\text{-max}})]^P$ will always fall between 0 and 1.0 for all values of P. With these quantities defined as such, the ratio of the quantities at values close to the respective maximum responses will be negligible. The rates are heavily weighted then by the magnitude of $(Y_i - Y_{i\text{-min}})$ the $(Y_j - Y_{j\text{-min}})$ terms.

The KC distance measure employs the same underlying additive transformation, but base their procedure on a distance function, which incorporates a variance–covariance matrix. This will result in two marginal rates of substitution: one for uncorrelated variables and one for correlated variables. Using (17), the marginal rates of substitution for any λ of $u(d_1, d_2, \ldots, d_n)$ for any given $i > 1$, $j > 1$, and $i \neq j$, are unique. Therefore, for the uncorrelated case, given, $\tau_\iota =$ target i, $i = 1$, and $j = 2$,

$$\lambda_{d_1,d_2} = \frac{\partial u_{d_1}(d_1, d_2, \ldots, d_n)}{\partial u_{d_2}(d_1, d_2, \ldots, d_n)} = \frac{\sigma_2^2(Y_1 - \tau_1)}{\sigma_1^2(Y_2 - \tau_2)} \tag{21}$$

What is seen in (21) is a ratio of the distance from the target for each response is multiplied by the ratio of the variance of the second response to the variance to the first response.

In the correlated case, the marginal rates of substitution for any λ of $u(d_1, d_2, \ldots, d_n)$ for any given $i > 1$, $j > 1$, and $i \neq j$, are unique. Therefore, for the correlated case, given, $\tau_\iota =$ target i, $i = 1$, and $j = 2$,

$$\lambda_{d_1,d_2} = \frac{\partial u_{d_1}(d_1, d_2, \ldots, d_n)}{\partial u_{d_2}(d_1, d_2, \ldots, d_n)} = \frac{\sigma_2^2[\sigma_{12}^2(Y_1 - \tau_1) + \sigma_1^2(Y_2 - \tau_2)]}{\sigma_1^2[\sigma_{12}^2(Y_2 - \tau_2) + \sigma_2^2(Y_1 - \tau_1)]} \tag{22}$$

This is similar to (21) but now includes the correlated elements. Table 3 displays the marginal rates of substitution for each of the response pairs. From the Del Castillo *et al.* (1996) example, response 1 and 4 are correlated, as well as 2, 3, 5, and 6 .

For the desirability method, all marginal rates of substitution are 1.0 as given in (18). The rates in the modified desirability model differ from (18) only in that responses 1 and 4 were weighted twice as much as responses 2, 3, 5, and 6. When comparing the tradeoffs calculated for the Global Criterion method it is seen that they differ in every case. These values are in bold.

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From Table 3 it should be noted that a number of the marginal rates of substitution are close to zero or deviate from zero. In the case of the Global Criterion and KC methods, this is due to the underlying trade-off relationship. It is seen in (20), (21), and (22) that the marginal rate of substitution is tied to the target value of the response of interest. When the response of interest is close to the target, the marginal rate of substitution is small or close to zero (e.g., pairs 1 to 2 for the Global Criterion method). Conversely, as the response of interest moves away from the target value, the marginal rate of substitution moves away from zero (e.g., pairs 4 to 5 for the Global Criterion and pairs 3 to 4 for the KC uncorrelated methods).

For the multiplicative methods, the response tradeoffs are either constant or based on the exponents chosen by the user. A user must consider the non-constant nature of the trade-offs between responses when using the Global Criterion method. The marginal rate of substitution for the KC distance method depends on two items: the correlation between responses and the magnitudes of the elements in the covariance matrix.

5. IMPLICATIONS WITH REGARD TO RISK PREFERENCE BEHIND THE UNIFYING METHODS

In the unifying objective approaches, the risk aversion of a decision maker is also implicitly defined. A distinct risk preference is associated with each transformation that is performed on a response. This section discusses how to measure risk aversion and derives the risk structures for the transformations used in each underlying desirability function. Four types of functions are presented and analyzed: a Gompertz or S-shaped function, a linear function, a convex function, and a concave function. Note that while the KC distance method does not specify a transformation, the implicit transform mirrors the Global Criterion transform for the uncorrelated case. For the modified desirability function these functions are obtained by adjusting the weights in (7) and (8). For example, by setting r, s, and/or t to 1.0 the linear function is realized. In setting r, s, and t to a value less than 1.0 but greater than 0.0 the concave function is realized, while setting r, s, and/or t to a value greater than 1.0 the convex function is realized.

Within utility theory, there exists different sets of theorems that define the risk aversion of a utility function $u(Y)$. Keeney and Raiffa [10] give the following definition as a measure of local risk aversion, $r(Y)$, for a utility function $u(Y)$,

$$r(Y) = \frac{u''(Y)}{u'(Y)} \tag{23}$$

Alternatively,

$$r(Y) = -\frac{\mathrm{d}}{\mathrm{d}x}[\log u'(Y)] \tag{24}$$

and define the following theorems with respect to $r(Y)$ for interpretation purposes:

Theorem 1. *The utility function u is risk neutral iff the associated risk aversion r is zero. A person is said to be risk neutral if they behave in the same manner regardless of the monetary value of the decisions.*

Theorem 2. *The utility function u is increasingly risk averse iff the associated risk aversion r is positive and increasing. A person who is increasingly risk averse behaves more and more conservatively as the monetary value of the decisions increase.*

Theorem 3. *The utility function u is increasingly risk prone iff the associated risk aversion r is negative and decreasing. A person who is increasingly risk prone tends to gamble more as the monetary value of the decisions increase.*

With this definition of local risk aversion and the subsequent theorems, the risk aversion implied within the transformations recommended by the underlying desirability functions can now be identified.

The desirability method uses the Gompertz growth curve transformation as defined in (4) with a linear function:

$$d_i = \exp(-(\exp - (b_0 + b_1 Y_i))) \tag{25}$$

where $b_0 > 0$, $b_1 > 0$, $Y_i > 0$, and $d_i = u(Y_i)$. Therefore, the local risk aversion, $r(Y_i)$, at Y_i is

$$r(Y_i) = -b_1 + b_1 \exp[-(b_0 + b_1 Y_i)] \tag{26}$$

Thus, $r(Y_i) < 0$ and decreases for all Y_i on $[0, Y_{i\text{-max}})$. $Y_{i\text{-max}}$ is the maximum acceptable value for response i. By Theorem 3, the utility function $u(Y_i)$ is increasingly risk-prone. When using the Gompertz transformation it is always assumed that the decision maker is increasingly risk-prone. In other words, the decision maker is willing to gamble more and more if the payoff is greater in each instance.

For the modified desirability method, allowing the exponents, $r = s = t = R$, and $\tau =$ target, the local risk aversion is as follows for the one-sided transform

$$r(Y_i) = \frac{(R-1)}{(Y_i - Y_{i\text{-min}})} \quad \text{for } Y_i \geq Y_{i\text{-min}} \tag{27}$$

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Table 4. Response settings for three underlying desirability transformations

Response	Transformation type: Risk neutral	Risk taking	Risk averse
Y_1	186.06	186.11	185.94
Y_2	174.49	174.67	174.64
Y_3	172.02	172.22	172.22
Y_4	192.63	193.30	192.79
Y_5	173.04	173.37	173.23
Y_6	184.95	185.17	185.21

and the two-sided transform

$$r(Y_i) = \begin{cases} \dfrac{(R-1)}{(\tau - Y_{i\text{-min}})} & \text{for } Y_i \le \tau \\ \dfrac{(R-1)}{(\tau - Y_{i\text{-max}})} & \text{for } Y_i \ge \tau \end{cases} \tag{28}$$

and therefore, for all cases, when $R > 1$, $r(Y_i) > 0$ and increases for all Y_i on $[0, Y_{i\text{-max}})$. $Y_{i\text{-max}}$ is the maximum acceptable value for response i. From Theorem 2, the utility function $u(Y_i)$ is increasingly risk-averse. However, when $R < 1$, $r(Y_i) < 0$ and decreases for all Y_i on $[0, Y_{i\text{-max}})$, Theorem 3 states that the utility function $u(Y_i)$ is increasingly risk-prone. In the case where $R = 1$, (27) and (28) is used and from Theorem 1, when $r(Y_i) = 0$ the utility function $u(Y_i)$ is risk neutral.

Therefore, a user can dictate the risk characteristics of the underlying desirability function by specifying the value of the exponent r in the transform. This concept addresses Derringer and Suich's suggestion that Harrington's transformations are not flexible in the sense that the functions cannot assume a variety of shapes. In varying the shape of the underlying desirability function, a decision maker assigns a level of risk preference.

In summary, when using the Gompertz transformation it is always assumed that the decision maker is increasingly risk-prone. For the modified desirability and global criterion transformations, the same assumptions regarding risk apply. For these transformations, the risk preference is based on the value of the exponent R used. In the case where $R > 1$, a decision maker is increasingly risk-averse, for $R < 1$ a decision maker is increasingly risk-prone, and for $R = 1$ a decision maker is risk-neutral.

Table 4 displays the response settings for the Del Castillo example using the desirability method and three different underlying transformations, risk neutral, risk taking, and risk averse. A user should be cognizant of the underlying transformation a method is using as it implicitly assigns risk preference. However, it can be seen for the Del Castillo example that the use of different underlying transformations has little effect on the response settings for this application. It must be noted that this is a result from only one application, and it is imperative that a user understands what the risk preference is of the underlying transformation being used.

6. SUMMARY

The idea of unifying objectives is to transform a multi-response design problem into a single response problem using mathematical transformations. Four unifying approaches are studied in this paper. Each of these methods contains assumptions regarding a user's risk preference, response relationship, and the marginal rate of substitution, and a user should understand these assumptions before implementing any of the methods.

In the desirability method, the Gompertz growth transformation assumes that the decision maker is increasingly risk-prone. It is also equivalent to an additive utility function. This fact has implications due to the response independence conditions of the additive utility function. Specifically, the model assumes that a user's utility depends only on the marginal probability distributions of the respective attributes and not on their joint probability distribution. Finally, the marginal rate of substitution for all attributes is equal. Therefore, it is assumed that the decision maker is willing to give an equal amount of one response to gain the same amount of another response.

In the case of the modified desirability method, the transformation assumes the following based on the exponent R.

- If $R > 1$ then the decision maker is assumed to be increasingly risk-averse.
- If $R < 1$ then the decision maker is assumed to be increasingly risk-prone.
- If $R = 0$ then the decision maker is assumed to be risk-neutral.

This method is also equivalent to the additive utility function and has the same implications as above. Finally, the marginal rate of substitution is a ratio of the exponential weights for each attribute in (19). This assumes the rates are constant but not all equal, and a decision maker has complete control over what those rates will be when assigning the exponents.

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The Global Criterion transformation leads to the same assumptions regarding a user's risk preference and response independence as the modified desirability method above. However, in the uncorrelated case of the KC method, we see an underlying linear transform raised to the second power, i.e. $R > 1$, and the decision maker is increasingly risk averse. The marginal rates of substitution for the Global Criterion approach are a function of the response itself. For the KC method, it is a function of either the variance structures in the uncorrelated case (see (21)) or the variance and covariance structures in the correlated case (see (22)).

In selecting a multiple response approach, the method predisposes the user's perspective with respect to risk preference, response independence, and response trade-offs. The user should be knowledgeable about their preferences as well as the assumptions behind the methods.

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