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Process capability: a criterion for optimizing multiple response product and process design

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In this research, we consider the maximization of process capability as the criterion in product/process design that is used for selecting *preferred* design factor levels and propose several approaches for single and multiple response performance measure designs. All of these approaches assume that the relationship between a process performance measure and a set of design factors is represented via an estimate of a response surface function. In particular, we develop: (i) criteria for selecting an optimal design, which we call *MCpk* and *MCpm*; (ii) mathematical programming formulations for maximizing *MCpk* and *MCpm*, including formulations for maximizing the desirability index (Harrington, 1965) and for maximizing the standardized performance criteria (Barton and Tsui, 1991) as special cases of the formulation for maximizing *MCpk*; (iii) formulations for considering cost when maximizing *MCpk* and *MCpm*; (iv) a means for assessing propagation of error; (v) a robust design method for assessing design factor effects on residual variance; (vi) a means for assessing the optimality of a proposed solution; and (vii) an original application in the screening of printed circuit board panels.

1. Introduction

The selection criteria with which the quality of many products and/or processes is assessed is an important consideration that greatly affects management decisions in world class manufacturing organizations. In product and process design numerous criteria have been proposed and studied for assessing the appropriate levels of design factors that influence one or more performance measures. These approaches broadly include: (i) maximizing process yield or the joint likelihood that performance measures are within design specifications (Barton and Tsui, 1991; Brayton *et al.*, 1981; Director and Hachtel, 1977; Plante, 1999); (ii) maximizing desirability of performance measures with respect to tolerances and nominal targets (Castillo *et al.*, 1996; Derringer and Suich, 1980; Harrington, 1965); (iii) minimizing, usually via a Taguchi (1986) quadratic loss function, the cost associated with a process (Ames *et al.*, 1997; Askin and Goldberg, 1988; Kacker, 1985; Pignatiello, 1993; Plante, 1999); (iv) minimizing the variance of a process performance measure, subject to the achievement of a nominal target for the average response, which is called a dual response (variance and mean) approach (Castillo and Montgomery, 1993; Vining and Myers, 1990); and (v) various partitioning procedures that attempt to partition the design factors into those that influence the mean and those that influence the variance (Elsayed and Chen, 1992; Hunter, 1985; Leon *et al.*, 1987). In one way or another, all of these approaches focus directly or indirectly on: (i) minimizing the deviation of a performance mean from target; and (ii) minimizing performance variance.

Over the last two decades, many world class manufacturing companies (Automobiles, Consumer Electronics, Chemicals, etc.) have made the transition *from* relying on process yield measures, such as composite yield, as an indicator of process performance *to* the assessment of process capability for monitoring process status as well as tracking process improvements. In particular, capability measures such as *Cpk* and *Ppk* have received wide acceptance as the criteria of choice with which to assess process performance (Harry, 1994; Rodriguez, 1992; Spiring, 1997; Udler and Zaks, 1997). Many organizations are striving to assess the process capability of each of their processes and achieve world class levels of at least 1.33 *Cpk* or better. It is well known and experience has often shown that the achievement of such levels of capability results in significantly lower costs, higher quality, greater productivity, etc.

As a process performance criterion, the *Cpk* process capability index is more sensitive to increased process variance than to the deviation of the process mean from specification limits. Thus, when it becomes necessary to accept a tradeoff between a lower variance process and one that is more centered on target, the process capability measure *Cpk* will tend to select optimal process designs

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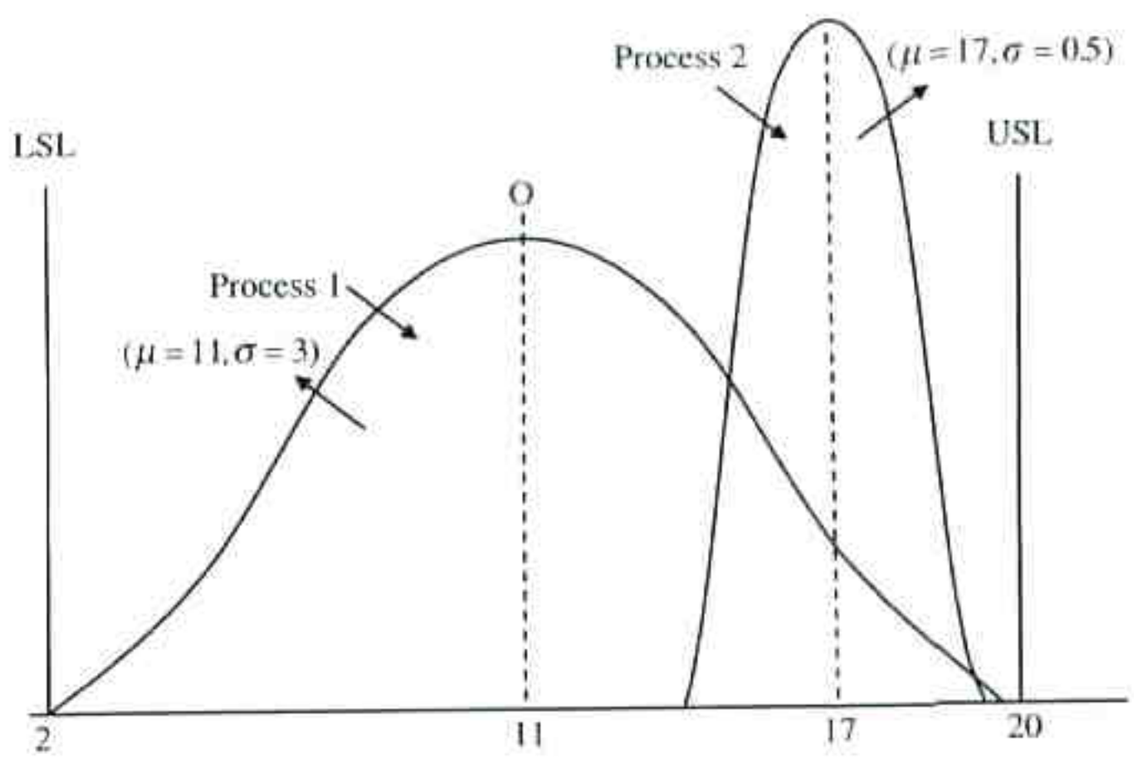
that are consistently off target, rather than inconsistently on target. Such a process environment is more conducive to the realization of further process improvement possibilities (Taguchi and Clausing, 1990). For example, consider the representation of two processes as shown in Fig. 1. As depicted in this figure, the expected cost of process 2 is four times that of process 1, but the process capability, via *Cpk*, of process 2 is twice that of process 1 (note: the *Cpm* process capability index would have selected process 1, since *Cpm* is essentially inversely related to the square root of the expected cost function). Although process capability did not select the low cost process, it did select the *preferred* process, having the greatest process potential of $Cp = 6$, which, if achieved would result in a true low cost process that is 36 times less costly than process 1. Thus, the *Cpk* process capability index tends to select those processes that are *preferred* for continuous learning and improvement. That is, the *preferred* processes for successful investigation of additional factors that may *significantly* affect the mean of a process performance measure. Succinctly, to continuously improve *Cpk*, one must continuously learn about and understand the effect of design factors on the mean and variance of process performance. The successful implementation of this understanding leads to continuous improvement of process performance.



$$Cost = \sigma^2 + (\mu - O)^2;\ Cpk = \mathrm{Min}\left\{\frac{(\mu - LSL}{3\sigma}, \frac{(USL - \mu)}{3\sigma}\right\};$$

$$Cpm = \frac{(USL - LSL)}{6\sqrt{\sigma^2 + (\mu - O)^2}}$$

Process	*Cost*	*Cpk*	*Cpm*
1	9.0	1.0	1.0
2	36.25	2.0	0.5
2 (Adjusted)	0.25	6.0	6.0

Fig. 1. A comparison of expected cost and process capability (*Cpk*, *Cpm*).

In this research, we consider the maximization of process capability as the criterion for selecting preferred design factor levels in product/process design. The proposed models assume that relationships between process performance measures and design factors are represented via an estimate of response surface functions. Via design of experiments such estimates are obtained via either center point designs, central composite designs, or Box–Behnken designs (Box and Behnken, 1960; Montgomery, 1997).

In Section 2 we use process capability and the concepts of desirability (Harrington, 1965) to develop capability measures for a process having multiple performance measures, which we call *MCpk* and *MCpm*. Nonlinear programming formulations are developed and proposed for determining the design factor levels that maximize *MCpk* and *MCpm*, respectively. We illustrate the formulation for maximizing *MCpk* via an example adapted from Young *et al.* (1991), which considers the impact of four design factors on nine performance measures in the design of a transistor in VLSI (Very Large Scale Integrated) electronic device design. To illustrate the accessibility of this technology to a wide range of potential users, the model is formulated in EXCEL and the "SOLVER" optimization package is used to obtain an optimal solution. A brief discussion of the GRG (Generalized Reduced Gradient) algorithm used in this spreadsheet add-on is presented by Castillo and Montgomery (1993).

In Section 3 we augment the formulations developed in Section 2 by accounting for two possible ways that design factors affect performance measure variances. The first way is via the propagation of error (transmitted variation) of the design factors. In this case, the design parameters are produced by processes that have some inherent variation. For the second way, the variance of the performance measure residuals (resulting from response surface estimation) is modeled as a nonlinear response function of the design factors.

In Section 4 we provide an example to demonstrate an implementation of our proposed procedures. The example considers the impact of seven design factors on five performance measures in the screening of printed circuit board panels. Finally, in Section 5, we summarize the research and consider opportunities for further research.

2. Formulations for maximizing *MCpk* and *MCpm*

The multiple performance product design problem consists of m performance measures that are influenced by a common set of n design factors. The relationship between the ith performance measure and the design factors is expressed as:

$$Y_i = a_{i0} + \sum_{j=1}^{n} a_{ij}X_j + \sum_{j=1}^{n} a_{ijj}X_j^2 + \sum_{k=1}^{n}\sum_{l>k}^{n} a_{ikl}X_kX_l + e_i \qquad i = 1, \ldots, m. \quad (1)$$

where,

Y_i = the ith performance measure;
ε_i = an error term that is independently distributed with variance $\sigma^2_{\varepsilon_i}$ and mean = zero;
a_{ij} = coefficients for the linear terms of the ith performance measure;
a_{ikl} = coefficients for the squared and interaction terms of the ith performance measure.

The expected value or mean of the performance measure is designated as μ_i, such that, assuming no variance in the design factors:

$$\mu_i = a_{i0} + \sum_{j=1}^{n} a_{ij}X_j + \sum_{j=1}^{n} a_{ijj}X_j^2 + \sum_{k=1}^{n}\sum_{l>k}^{n} a_{ikl}X_kX_l \qquad i = 1, \ldots, m. \quad (2)$$

The ith performance measure has engineering specifications limits or tolerances that are designated LS_i for the lower specification limit and US_i for the upper specification limit. We propose a system capability index of the process called $MCpk$ which is the geometric mean of performance measure Cpk's, and will be used as the criterion to assess multiple response product/process designs. We express $MCpk$ as a function of the design factors $X = (X_1, X_2, \ldots, X_n)$ as follows:

$$MCpk = \left\{ \prod_{i=1}^{m} \mathrm{Min}\left[\frac{(\mu_i - LS_i)}{3\sigma_i}, \frac{(US_i - \mu_i)}{3\sigma_i} \right] \right\}^{(1/m)}, \quad (3)$$

where

σ_i = the standard deviation of the ith error term, σ_{ε_i}.

Note that when $m = 1$, this is just the Cpk index.

The fundamental philosophy underpinning the $MCpk$ index has profound implications in quality. That is, if any one or more performance measures is so unsuitable to their respective tasks (i.e., close to zero capability) then the overall product is deemed unacceptable, regardless of the capability levels of the remaining performance measures. Indeed the $MCpk$ measure is strongly influenced by those performance measures possessing small capability. Further, this criterion lends itself very well to planning continuous improvement strategies, since the individual performance capabilities illuminate where process/product design improvement can have the greatest impact. Additionally, the $MCpk$ criterion is more universally applicable, since it does not require a target value for each performance measure.

The general nonlinear programming formulation for determining the levels of the design factors that maximize $MCpk$ is then,

$$\text{Maximize}_X: \prod_{i=1}^{m} C_i, \quad (4a)$$

subject to

$$\frac{(\mu_i - LS_i)}{3\sigma_i} \geq C_i \quad i = 1, \ldots, m, \quad (4b)$$

$$\frac{(US_i - \mu_i)}{3\sigma_i} \geq C_i \quad i = 1, \ldots, m, \quad (4c)$$

$$C_i \geq 0 \quad i = 1, \ldots, m. \quad (4d)$$

and

$$LR_j \leq X_j \leq UR_j \quad j = 1, 2, \ldots, n. \quad (4e)$$

Note that (4b) and (4c) are essentially chance constraints, where $3C_i$ represents the minimum multiple of standard deviations that the ith performance measure is from specification limits. Further, LR_j and UR_j represent lower and upper range constraints on the jth design parameter. Such constraints may represent physical limits or may also represent the experimental range for X_j used in obtaining an estimate for Equation (1). Given a solution to problem (4), the $MCpk$ capability measure is then obtained as follows:

$$MCpk = \left(\prod_{i=1}^{m} C_i \right)^{(1/m)}. \quad (5)$$

Given the relationship of the Cpm process capability index to quality cost (Fig. 1), we also consider an additional criterion. Similar to the $MCpk$ index, this criterion is the geometric mean of the performance measure Cpm's. This is called the $MCpm$ capability criterion, which is expressed as follows:

$$MCpm = \left\{ \prod_{i=1}^{m} \left[\frac{(US_i - LS_i)}{6\sqrt{[\sigma_i^2 + (\mu_i - O_i)^2]}} \right] \right\}^{(1/m)}, \quad (6)$$

where

O_i = the target or objective value for the ith performance measure.

However, as we mentioned previously, the $MCpm$ index, like the Cpm index, requires targets for each performance measure, and is thus not as flexible as the $MCpk$ criterion.

A general nonlinear programming formulation for maximizing $MCpm$ can be expressed as,

$$\text{Maximize}_X: \prod_{i=1}^{m} Cpm_i \quad (7)$$

subject to (4e).

Given a solution to problem (7), the $MCpm$ capability measure is then obtained as follows,

$$MCpm = \left(\prod_{i=1}^{m} Cpm_i \right)^{(1/m)}. \quad (8)$$

In addition, several other system capability measures may also be developed via an appropriate modification of (4).

For example, a $MCpkm$ criterion would be developed by replacing the denominators in (4b) and (4c) with $3\sqrt{\sigma_i^2 + (\mu_i - O_i)^2}$. The resulting $MCpkm$ criterion would then be assessed using (5). Such a criterion would account for off-center processes in a manner similar to $MCpk$, but, like $MCpm$, requires target values for the performance measures.

2.1. *Special cases of the MCpk formulation*

A criterion called the desirability index was proposed by Derringer and Suich (1980) for optimizing multiple performance designs. The formulation in (4) for $MCpk$ can be easily adapted for the desirability index. This can de accomplished by replacing the denominators in (4b) and (4c) with $(O_i - LS_i)$ and $(US_i - O_i)$, respectively. For two-sided specification limits, the target value, O_i, is usually the midpoint between the lower and upper limits. For one-sided limits, a reasonable guess is required as an estimate for the target value. For the one-sided limit case, Derringer and Suich (1980) suggested that a target be selected such that exceeding the target is judged to provide no further benefit. Further, especially for one-sided limits, (4d) is modified by $0 \le C_i \le 1$, where the upper limit of one suggests that there is no additional improvement once a target has been reached.

The standardized performance criterion was proposed by Barton and Tsui (1991) as an approximation to yield, which is the joint likelihood that the performance measures are within tolerance. The standardized performance criterion places emphasis on the performance measure that has the least capability. This criterion essentially focuses on the weakest link in the chain or $MCpk$ of capability. The standardized performance criterion can be stated as:

$$SP = \text{Maximize}_X \left\{ \text{Minimum}_Y \left[\text{Minimum} \left(\left(\frac{\mu_i - LS_i}{\sigma_i} \right), \left(\frac{US_i - \mu_i}{\sigma_i} \right) \right) \right] \right\}. \tag{9}$$

The standardized performance criterion can be easily formulated as a special case of problem (4). This can be accomplished by replacing each occurrence of C_i in the constraints of problem (4) with C, and replace the objective in (4a) with Maximize_X: C, as follows:

$$\text{Maximize}_X: C, \tag{10a}$$

subject to

$$\frac{(\mu_i - LS_i)}{3\sigma_i} \ge C \quad i = 1, \ldots, m, \tag{10b}$$

$$\frac{(US_i - \mu_i)}{3\sigma_i} \ge C \quad i = 1, \ldots, m, \tag{10c}$$

$$C \ge 0, \tag{10d}$$

and

$$LR_j \le X_j \le UR_j \quad j = 1, 2, \ldots, n. \tag{10e}$$

2.2. *Importance considerations*

At times, there may be a requirement to account for an additional tradeoff among performance measures, other than $MCpk$ or $MCpm$. Especially when nonconformance by a performance measure or measures incurs substantially larger costs. For example, repair costs could differ for some performance measures, and still others may require the complete scrapping or rework of the product. Consequently, it may be more important to achieve a higher capability on one or more measures at the expense of others. An approach that accounts for the relative importance of performance measures has been developed for the desirability index, and we offer a similar one here.

Let R_i represent the nonconformance costs (repair/rework/replacement costs) associated with the ith performance measure, and let $R_{\min} > 0$ represent the smallest of these costs over all performance measures. The mathematical programming formulation in (4) is then modified by replacing (4b) and (4c) with:

$$\left[\frac{(\mu_i - LS_i)}{3\sigma_i} \right]^{(R_i/R_{\min})} \ge C_i \quad i = 1, \ldots, m, \tag{11a}$$

$$\left[\frac{(US_i - \mu_i)}{3\sigma_i} \right]^{(R_i/R_{\min})} \ge C_i \quad i = 1, \ldots, m. \tag{11b}$$

Following a solution to problem (4) with revised constraints as shown in (11a) and (11b), $MCpk$ can be obtained as follows,

$$MCpk = \left[\prod_{i=1}^{m} C_i^{\left(\frac{1}{R_i}\right)} \right]^{(R_{\min}/m)}. \tag{12}$$

Note that if costs differ for both the upper and lower specification limits, then we can replace R_i with R_{LS_i} in (11a) and with R_{US_i} in (11b).

2.3. *Example: VLSI device design*

This example is adapted from a VLSI device design problem considered by Young *et al.* (1991). They discussed an application to a BIMOS NPN transistor. In particular, a detailed study of the polyemitter device in an NPN transistor was presented. This problem consists of four design factors: (i) base dose (X_1); (ii) base energy (X_2); (iii) well dose (X_3); and (iv) well drive time (X_4). There are nine performance measures: (i) emitter-current gain (Y_1) with a lower specification limit of 50; (ii) collector–base capacitance at zero bias (Y_2) with an upper specification limit of 4.93; (iii) collector–base capacitance at a 5 volt bias (Y_3) with an upper specification limit of

1.79; (iv) emitter–base capacitance at zero bias (Y_4) with an upper specification limit of 3.61; (v) knee-current (Y_5) with a lower specification limit of 0.53; (vi) collector-resistance (Y_6) with an upper specification limit of 730; (vii) base-resistance (Y_7) with an upper specification limit of 1220; (viii) peak cutoff frequency (Y_8) with a lower specification limit of 4.2; and (ix) propagation delay (Y_9) with an upper specification limit of 155. Notice, that each of these performance measures has only a one-sided specification limit. Thus, without targets for the performance measures, we will not use this example to illustrate the $MCpm$ index. We will, however, be able to demonstrate the use of this index in Section 4. The estimated response surface functions for each of the performance measures are (using coded values for the design factors).

Emitter-current gain: $\mu_1 = 68.14 - 24.8X_1 - 11.6X_2 + 9.02X_1^2 + 6.27X_2^2$, with variance $\sigma^2_{\varepsilon_1} = 6.812$.

Collector–base capacitance with a 0 Volt bias: $\mu_2 = 4.457 - 0.102X_2 + 0.385X_3$, with variance $\sigma^2_{\varepsilon_2} = 0.446$.

Collector–base capacitance with a 5 Volt bias: $\mu_3 = 1.548 - 0.0658X_2 + 0.172X_4$, with variance $\sigma^2_{\varepsilon_3} = 0.154$.

Emitter–base capacitance with a 0 Volt bias: $\mu_4 = 2.649 + 0.298X_1 - 0.651X_2$, with variance $\sigma^2_{\varepsilon_4} = 0.265$.

Knee-current: $\mu_5 = 0.7813 + 0.0661X_1 + 0.0302X_3 + 0.155X_4$, with variance $\sigma^2_{\varepsilon_5} = 0.0784$.

Collector-resistance: $\mu_6 = 415.4 - 29.6X_2 - 17.1X_3 - 224X_4 + 11.1X_2^2 + 64.2X_4^2$, with variance $\sigma^2_{\varepsilon_6} = 41.538$.

Base-resistance: $\mu_7 = 870.3 - 222X_1 - 51X_2 + 62.7X_1^2$, with variance $\sigma^2_{\varepsilon_7} = 87.03$.

Peak-cutoff frequency: $\mu_8 = 8.878 - 1.35X_1 - 2.75X_2 + 0.714X_4 + 0.708X_2^2 - 0.572X_4^2$, with variance $\sigma^2_{\varepsilon_8} = 0.887$.

Propagation delay: $\mu_9 = 137.39 - 7.83X_1 + 4.42X_2 + 1.83X_3 + 3.08X_1^2 + 2.95X_2^2$, with variance $\sigma^2_{\varepsilon_9} = 13.742$.

The VLSI problem was formulated using (4). The solution was $X = (-0.242, -0.615, -1, 0.569)$ with individual capabilities $Cpk_Y = (4.363, 0.397, 0.157, 0.409, 0.349, 19.748, 9.328, 2.542, 1.771)$ and a resulting $MCpk$ of 1.47, which, as previously suggested, is the same solution, in terms of design factors, that could be obtained by maximizing the desirability index with appropriately specified targets (i.e., targets that are at least as far into the tolerance regions as the results obtained via $MCpk$).

The standardized performance criterion was also formulated using (10). The solution was $X = (1, -0.339, -0.352, -0.427)$ with individual capabilities $Cpk_Y = (0.895, 0.286, 0.286, 0.286, 0.286, 9.825, 17.57, 1.391, 2.078)$ and a resulting $MCpk$ of 1.13. Clearly, the low capability of 0.157 resulting from maximizing $MCpk$ via (4) has been increased to a Cpk of 0.286. However, this was at the expense of reducing the capabilities of virtually all the other performance measures.

Finally, to assess the impact of different cost implications among the performance measures, the following nonconformance costs were used $R_Y = (0.5, 1, 1, 0.25, 0.5, 1, 1, 0.25, 0.25)$. Equations (11a) and (11b) replaced (4b) and (4c) in the formulation of the VLSI problem. The solution was $X = (0.288, -0.792, -1, 0.4)$ with individual capabilities $Cpk_Y = (3.176, 0.388, 0.191, 0.233, 0.36, 17.92, 13.15, 2.514, 1.999)$ and a resulting $MCpk$ of 1.419. Note that when costs are accounted for, the individual capabilities change, but that the direction of change is somewhat inconsistent with what would be expected under the influence of the given relative costs. For example, the capability of the first performance measure went from 4.363 to 3.176, having a relative cost of 0.5. Whereas the capability of the ninth performance measure improved from 1.771 to 1.999, having a lower relative cost of 0.25. Further, the maximum $MCpk$ was reduced from 1.47 to 1.419.

3. Effect of design parameters on performance measure variances

In this section we consider how design parameters affect performance measure variation. We first develop a procedure for modeling the effect of variation in design parameter processes that is transmitted to performance measures (Fathi, 1991). Next, we estimate the variance of the performance measures via a response surface function of the design parameters. This is similar, in principle, to the dual response method proposed by Vining and Myers (1990) for a single performance measure. Our approach, however, utilizes concepts of iterative reweighted least squares, which is typically used to model residual variances for a heteroscedastic system. When we augment (4) with the effects of design factors on performance measure variances, there is no guarantee that a solution to (4) is optimal. Thus, in this section, we also describe a procedure for obtaining reasonably good or capable solutions.

3.1. *Propagation of error*

We now consider that the design factors may come from processes that have some inherent variation. The transmission of this variation to performance is called error propagation. We assume that the design factors are independent with process means μ_{X_j} and variance $\sigma^2_{X_j}$, $j = 1, \ldots, N$. Via propagation of error the variance of a performance measure can be expressed as (equivalent to Fathi, 1991):

$$\sigma_i^2 = \sigma^2_{\varepsilon_i} + \sum_{j=1}^{n} \left\{ \sigma^2_{X_j} \times E\left[\left(\frac{\partial Y_i}{\partial X_j} \right)^2 \right] \right\}, \quad i = 1, \ldots, m \tag{13a}$$

where, using (1)

$$E\left[\left(\frac{\partial Y_i}{\partial X_j}\right)^2\right] = \left(a_{ij} + 2a_{ijj}\mu_{X_j} + \sum_{k\neq j}^{n} a_{ijk}\mu_{X_k}\right)^2 + \sum_{k\neq j}^{n} a_{ijk}^2\sigma_{X_k}^2 + 4a_{ijj}^2\sigma_{X_j}^2, \tag{13b}$$

and with a corresponding mean of

$$\mu_i = a_{i0} + \sum_{j=1}^{n} a_{ij}\mu_{X_j} + \sum_{j=1}^{n} a_{ijj}\left(\sigma_{X_j}^2 + \mu_{X_j}^2\right) + \sum_{k=1}^{n}\sum_{l>k}^{n} a_{ikl}\mu_{X_k}\mu_{X_l}. \tag{14}$$

It is clear from (13) and (14) that both the process means and variances of the design factors affect not only the means of the performance measures, but also their variances.

3.2. *Identification of design factors affecting variance*

An examination of the plots of the residuals against each design factor may reveal that some of the factors have an influence on performance measure variances (Montgomery, 1997). Identifying and accounting for such factors would clearly help to improve the resulting *MCpk* of a process design. In this regard, we propose the following robust design procedure for accounting for that part of the variation in the residuals that can be explained by the design factors. The fundamental idea is to partition the error variance $\sigma_{E_i}^2$ into explained and unexplained segments. This proposed procedure, for the ith performance measure, is as follows.

Step 1. Estimate the response surface function for the ith performance measure, retaining only significant factors and store the residuals from the final estimate.

Step 2. Consider an estimate of a response function for the squared residuals. This provides the mean square residual or variance of the ith performance measure as a response function of the design factors. In such cases, however, a better fit is often possible by estimating the absolute value of the residuals, as follows:

$$|\hat{e}_i| = b_{i0} + \sum_{j=1}^{n} b_{ij}X_j + \sum_{j=1}^{n} b_{ijj}X_j^2 + \sum_{k=1}^{n}\sum_{l>k}^{n} b_{ikl}X_kX_l. \tag{15}$$

or, under propagation of error

$$|\hat{e}_i| = b_{i0} + \sum_{j=1}^{n} b_{ij}\mu_{X_j} + \sum_{j=1}^{n} b_{ijj}\left(\sigma_{X_j}^2 + \mu_{X_j}^2\right) + \sum_{k=1}^{n}\sum_{l>k}^{n} b_{ikl}\mu_{X_k}\mu_{X_l}, \tag{16}$$

where

$\hat{e}_i$ = the fitted or predicted residual for the ith performance measure;
b_{i0} = response surface estimate of the intercept;
b_{ij} = response surface estimate of the main effect coefficient for the ith performance measure; and
b_{ikl} = response surface estimate of the square and interaction effect coefficients for the ith performance measure.

Then the estimated variance is:

$$\sigma_{e_i}^2(\mu_X) = |\hat{e}_i|^2. \tag{17}$$

Step 3. Clearly, (17) does not often account for or explain all of the residual variance associated with the ith performance measure. Thus, some portion of the residual variance remains unexplained. To account for this unexplained variance, we recommend determining that fraction of the residual variance that is not explained, via the coefficient of multiple determination as follows, for N data observations on the ith performance measure,

$$R_i^2 = \frac{\sum_{k=1}^{N} |\hat{e}_k|^2}{\sum_{k=1}^{N} e_k^2}, \tag{18}$$

where

R_i^2 = the coefficient of multiple determination for the residual variance;
e_k^2 = the kth squared residual from the response surface estimate for the ith performance measure.

The partition of the original error variance into explained and unexplained portions, for the ith performance measure, is then approximated as follows,

$$\sigma_{P_i}^2 \cong \sigma_{e_i}^2(\mu_X) + (1 - R_i^2)\sigma_{E_i}^2, \tag{19}$$

where,

$\sigma_{P_i}^2$ = an approximate partition of the error variance for the ith performance measure.

The expression that we use in (19) provides an effective approximation for partitioning the error variance. Here, the overriding benefit is that, when optimizing, this partitioning of the error variance provides at least a reasonable insight with respect to the levels of the design factors that tend to minimize (19) and thereby increase *MCpk* and *MCpm*.

Including the propagation of error (19), the variation of the ith performance measure can now be expressed as,

$$\sigma_i^2 = \sigma_{P_i}^2 + \sum_{j=1}^{n}\left\{\sigma_{X_j}^2 \times E\left[\left(\frac{\partial Y_i(X)}{\partial X_j}\right)^2\right]\right\} \quad i = 1, \ldots, m. \tag{20}$$

3.2.1. *An example in injection molding*

We tested the proposed robust design procedure on a well known problem from Montgomery (1997). In this problem there is only one performance measure, which is the shrinkage (Y) of a plastic part that is produced from an injection molding operation. There are six design factors that were expected to influence shrinkage. These factors are: mold temperature (X_1); screw speed (X_2); holding time (X_3); cycle time (X_4); gate size (X_5); and holding pressure (X_6). A 2^{6-2} fractional factorial design, with a single replication was used to gather data for this problem. Since all of the design factors are quantitative, we estimated the following response functions (since there is confounding among the interactions, we chose, as did Montgomery (1997), those two-way interactions which contained terms that were also among the most significant main effects. Of course, such choices are tentative by nature and would have to be validated by further experimentation, or at least via validation experiments of the predictions from our analysis.),

$$\mu = 27.313 + 6.938\mu_{X_1} + 17.812\mu_{X_2} + 5.937\mu_{X_1}\mu_{X_2},$$

with variance, $\sigma_\varepsilon^2 = 20.73$, and partitioned error variance:

$$\sigma_P^2 = 0.02\sigma_\varepsilon^2 + (3.25 - 0.25\mu_{X_1} + 1.875\mu_{X_3} + 0.4375\mu_{X_4} - 0.25\mu_{X_5} - 0.375\mu_{X_1}\mu_{X_3} + 0.375\mu_{X_3}\mu_{X_4} + 0.75\mu_{X_4}\mu_{X_5})^2.$$

Using an upper specification of 10 for shrinkage, the levels of the design factors that maximized Cpk are $X = (-1, -1, -1, -1, 1, 0)$, with a resulting Cpk of 0.748, and predicting an average shrinkage of 8.5 with a standard deviation of 0.668. Comparatively, using residual graphs and individual F tests for variance effects, Montgomery (1997) recognized only the holding time (X_3) as having a significant effect on residual variance. In this case, the partitioned error variance would be:

$$\sigma_P^2 = 0.094\sigma_\varepsilon^2 + (3.25 + 1.875\mu_{X_3})^2.$$

The optimal settings for the design factors for this case are $X = (-1, -1, -1, 0, 0, 0)$, with a resulting Cpk of 0.255, and predicted average shrinkage of 8.5 with a standard deviation of 1.962. Thus, at least in this injection molding problem, our proposed robust design procedure compares very favorably with that described in Montgomery (1997).

3.3. *Alternative solutions for maximizing MCpk and MCpm*

When propagation of error and robust design effects on performance variance are used in (4) we cannot guarantee that an optimal solution is indeed a global, versus local, optimal solution. Consequently, we propose a procedure for finding a "good" if not optimal solution. A by-product of this approach is that alternative solutions are often uncovered. Alternative solutions are those solutions that have the same maximum $MCpk$ or $MCpm$, but differ in the recommended means for the design parameters. Such alternative solutions would provide a decision maker more flexibility, or at least more choices, in arriving at a decision.

A standard recommendation for ill-behaved optimization functions is to initiate the optimization algorithm or search procedure with various initial starting values for the design parameters. If the same solution is arrived at from many different starting conditions, then it is highly likely that the solution obtained is the optimal solution. The fundamental questions are: how many and which set of points should be used as initial starting values? Serendipitously, and unlike many other such problems in optimization, we already have an excellent set of candidate initial values. We propose that the set of initial values contain the experimental design points that were originally used to estimate the response functions for each of the performance measures.

4. An application in the screening of printed circuit board panels

In this section we consider the optimization of a screen printing process that applies an etch-resist ink to a panel that is composed of a copper-clad laminate, which is actually copper foil applied at a thickness of 1 ounce of copper per square foot. The etch-resist ink is printed on the panel by passing a squeegee over a screen which contains a template of the desired circuit image.

After the circuit image is printed on the copper laminate, the unwanted copper is etched away using a cupric chloride solution. The purpose of the etch-resist ink is to protect the circuit copper from the etching process. After the unwanted copper has been removed, the etch-resist ink has served its purpose and is stripped away using a solution of caustic soda. The final product is an electrical circuit composed of: (i) copper pads where electronic components will be attached; and (ii) copper lines that will connect the electronic components. Figure 2(a and b) show enlarged illustrations of a very small part of an etch-resist ink printing, including possible defects that can occur during the screening process.

The size of the image that is printed on the panel is a critical factor. If the image is either too large or too small, substantial problems in component assembly are encountered. After the copper image is printed and etched, the board receives a hole punching operation which pierces holes in the board for the assembly of through hole electronic components. The tooling used to place the holes in the board is made of hardened steel, and is fabricated to place a component hole in the exact center of a copper pad. If the image size is not nominal (too large or too small), then the hole will not be centered on the copper pad. This means that there will be less copper

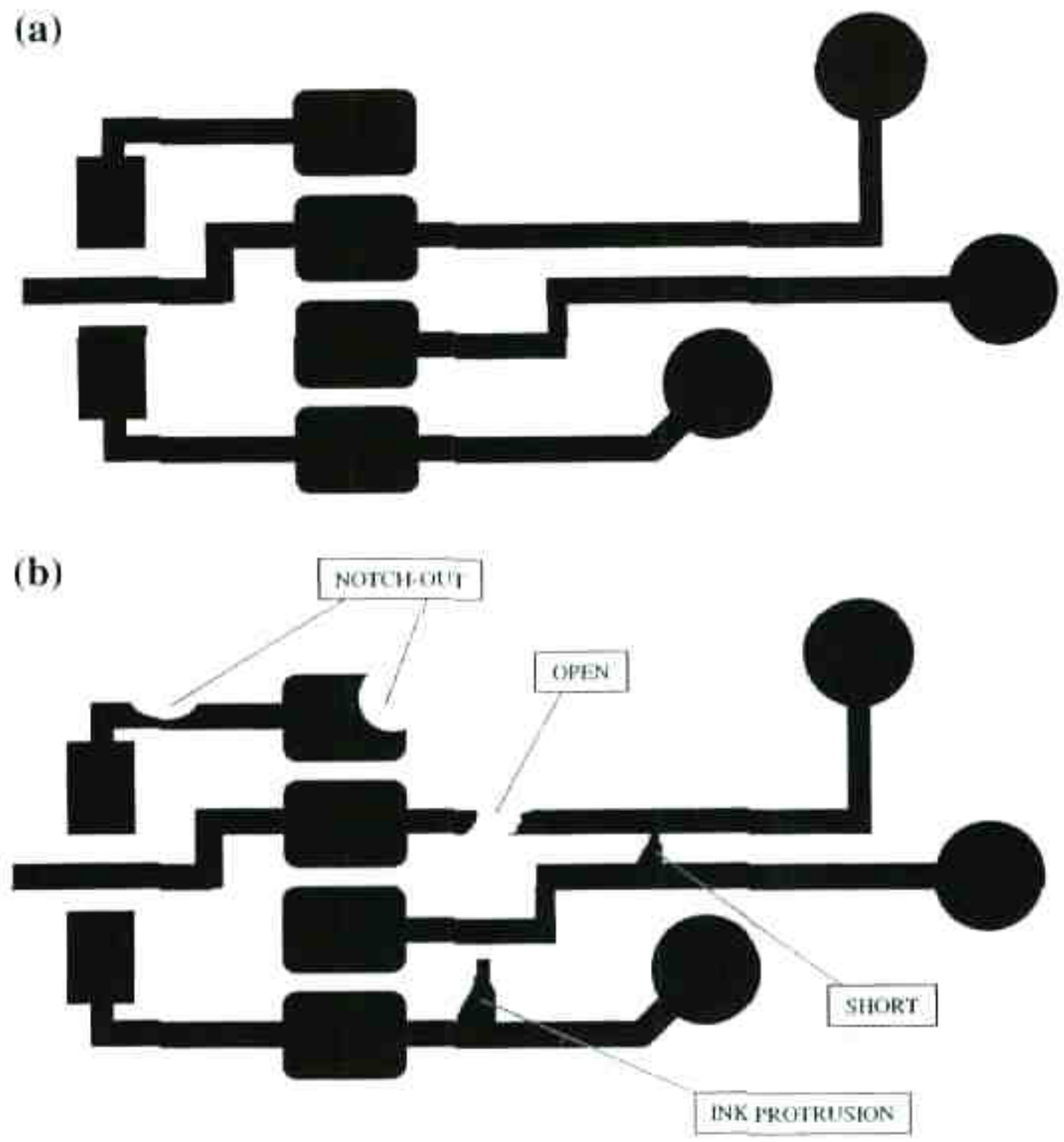


Fig. 2. (a) Acceptable print and (b) print defects.

on one side of the hole and more copper on the other side. The side with less copper will not be able to bond as well to a component lead when the board goes through the soldering process in component assembly. This condition also increases the chance of insufficient solder on a solder joint. These conditions lead to reliability problems in customer usage of the final product.

Another major problem created by an image size that is either too large or too small relates to surface mount components that are placed by automatic component insertion (ACI) equipment (SMD or Surface Mount Device machines) during component assembly. This surface mount assembly equipment relies on pilot holes, which have been punched in the board prior to screening, to locate the board on the machine and provide a two dimensional coordinate reference for component placement. If the printed image is not nominal, then the location of a copper pad with respect to the hole will vary. This will result in the placement of a surface mount electronic component that is not centered on a copper pad. The resulting lack of copper on one side of the surface mount component will result in insufficient solder on that side and compromise the strength of the solder joint, resulting in severe reliability problems and a commensurate degrading of quality reputation for the product.

4.1. *Measuring image size*

There are four measurements that measure the overall image size of a screen printing. Figure 3 provides an

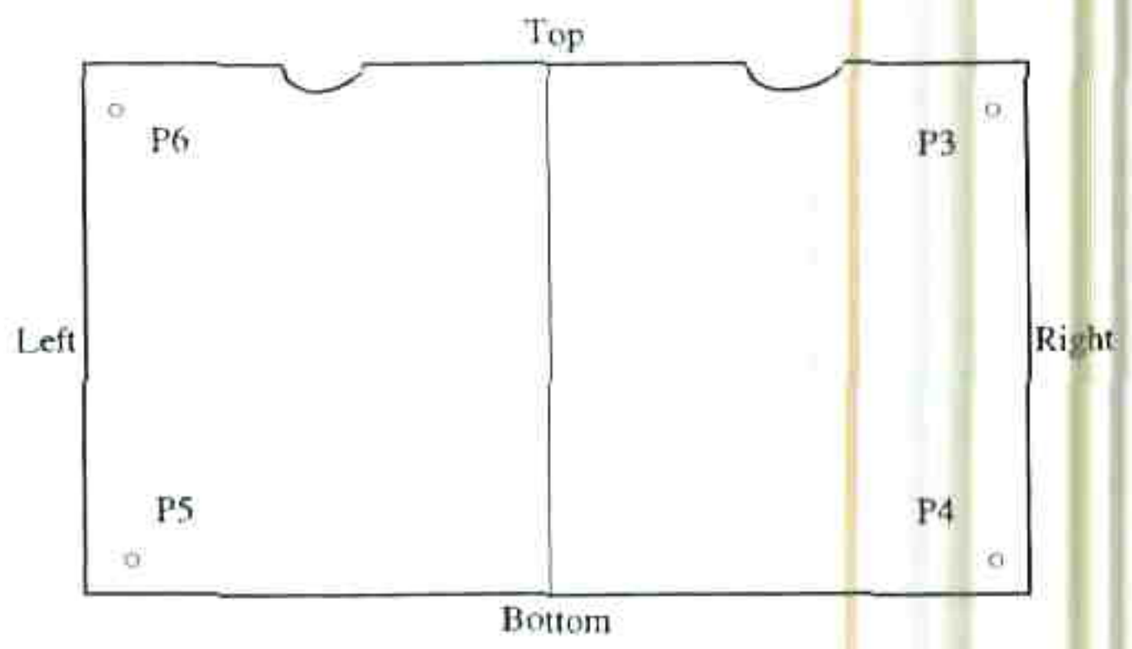


Fig. 3. Two-up panel.

illustration of a panel prior to screening. The illustrated panel is called a two-up panel, since a cluster of two identical circuit images will be simultaneously printed side-by-side on the same panel. The notches at the top of the board provide a poke-a-yoke (fool proof) mechanism for distinguishing the top of the panel from the bottom. This illustration also depicts four pilot holes, labeled P3, P4, P5 and P6. Also shown are four dimensions labeled top, bottom, left, and right, which will be used to assess image size.

During the screening operation four sets of tick marks are printed about each pilot hole as shown in Fig. 4 for pilot hole P3.

Using an X/Y coordinate measurement machine, an operator places the cross hairs, aided by a shaded circle, above and centered on a pilot hole, and enters the location, via electronic input, into a computer. The operator then centers the cross hairs in the printed tick marks, and again enters the location. The difference between the hole centered location, which is nominal, and the tick mark centered location provides two important deviations for measuring the deviation of image size from nominal. These deviations are the deviation in the Y coordinate and the deviation in the X coordinate, which we denote as ΔY_{P3} and ΔX_{P3}, respectively for pilot hole P3. Similar

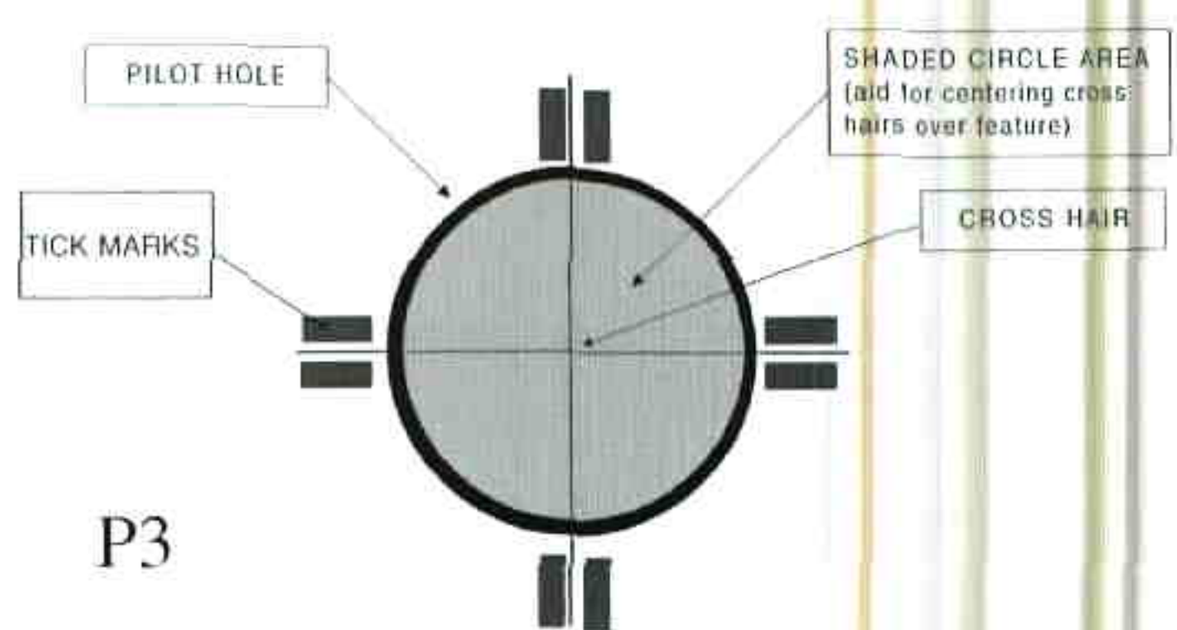


Fig. 4. Cross hair centered over pilot hole P3. When cross hair is centered over tick marks (from printing), there will be a positive deviation in the X coordinate.

measurements are obtained for the other pilot holes. Using these deviations, the top, bottom, left and right deviations of image size from nominal are obtained as follows,

$$\text{top deviation} = \Delta X_{P6} - \Delta X_{P3};$$

$$\text{bottom deviation} = \Delta X_{P5} - \Delta X_{P4};$$

$$\text{left deviation} = \Delta Y_{P5} - \Delta Y_{P6};$$

$$\text{right deviation} = \Delta Y_{P4} - \Delta Y_{P3}.$$

For a given dimension, a negative deviation implies that the image size is large with respect to that dimension. Whereas a positive deviation implies that the image size is small. The tolerance for each of these deviations is ±0.015 mm for each of the top (Y_1), bottom (Y_2), left (Y_3), and right (Y_4) dimensions.

4.2. *Design factors that influence image size*

There are seven design factors that influence the four dimensions of image size. The first three factors concern the screen. The screen is in a fixed position and a panel is placed under it for printing. The height of the screen above the panel can be adjusted by four screws. Two adjustment screws in the front can vary the Front Off Contact height, which is the first design factor with lower and upper experimental design limits of 2 and 6 mm (X_1). Two adjustment screws in the back of the screen are used to vary the Rear Off Contact height, which is the second design factor with limits of 2 and 6 mm (X_2). The third design factor is the height at which the screen peels off from the panel immediately after the squeegee passes, which can be adjusted and is called Peel Off with experimental design limits of 0 and 8 mm (X_3).

The remaining four design factors are concerned with the squeegee. The fourth design factor is called Skew. When the squeegee traverses across a panel, it can do so in a manner that is perpendicular to the panel, or via rotation about a vertical axis at the center of the squeegee, slightly skewed to the panel. The experimental design limits for this factor are −1 and 1 cm (X_4). There are two adjustment screws located on the top and at both ends of a rail that holds the squeegee. These screws can be used to adjust the downward pressure of the squeegee as it impinges on the screen and panel. This fifth design factor is called the Squeegee Pressure with limits −1 turn (gauged) and +1 turn (X_5). The sixth design factor is the angle that the squeegee makes with respect to the screen and panel. This factor is called the Squeegee Angle with limits of 63° and 83° (X_6). The final and seventh factor is the speed of the squeegee as it travels over the screen and panel, which is called the Squeegee Speed with limits of 100 and 400 mm/sec (X_7).

A 2^{7-3} (resolution IV) fractional factorial center-point design with three replications and four blocks was used to initially assess the impact of the seven design factors on each of the four performance measures. Each of the 60 printed panels (12 replications of the center point – three replications of each center point per block) was accorded an experimental design number and stored following the experiment. This was done in the event that the analysis showed significant outliers, upon which the corresponding panels would be remeasured to assure accuracy in the data. Using this approach, 12 panels were remeasured in this study. Further, the storage of the panels used in the experiments also allowed each panel to be more carefully examined for each of the defects depicted in Fig. 1b. Using a scale of one to 10, with 10 being the highest quality print, and seven representing a screen printing having a minimally acceptable quality (i.e., a lower specification limit of seven with a target of 10), a fifth performance measure was introduced called Print Quality (Y_5). This performance measure was added to assure that the optimal settings of the design factors for image size did not result in poor print quality. By simultaneously optimizing for print quality as well as image size, such a circumstance would be avoided.

Initial analysis of the data revealed that there were significant curvature effects for bottom deviation (Y_2), right deviation (Y_4), and Print Quality (Y_5). Thus, to assess curvature effects, 14 face-center experiments with three replications were conducted. A response surface function was estimated for each of the performance measures. Although there was confounding among the interactions, we selected, when significant, those interactions associated with the most significant main effects. Although this is usually a reasonable practice to follow (Montgomery, 1997), such assumptions should and have been validated via an implementation of the experimental/optimal results suggested as a result of this research.

4.3. *The estimated response functions*

The estimated response functions for each of the performance measures in the screening of printed circuit board panels are then,

Top Deviation:

$$\begin{aligned}\mu_1 = {} & 0.004\,57 + 0.002\,25\mu_{X_2} + 0.002\,54\mu_{X_3} - 0.001\,16\mu_{X_5} \\ & + 0.001\,06\mu_{X_6} + 0.001\,01\mu_{X_7} - 0.000\,83\mu_{X_2}\mu_{X_5} \\ & + 0.001\,05\mu_{X_2}\mu_{X_6} + 0.001\,51\mu_{X_2}\mu_{X_7}.\end{aligned}$$

With a variance, $\sigma^2_{\varepsilon_1} = 0.000\,005$, and partitioned error variance:

$$\begin{aligned}\sigma^2_{P1} = {} & 0.262\sigma^2_{\varepsilon_1} + (0.000\,69 + 0.000\,88\mu_{X_1} + 0.000\,66\mu_{X_2} \\ & - 0.000\,85\mu_{X_4} - 0.000\,28\mu_{X_6} + 0.0011(\mu^2_{X_5} + \sigma^2_{X_5}) \\ & + 0.000\,71\mu_{X_1}\mu_{X_2} - 0.000\,74\mu_{X_1}\mu_{X_4} - 0.000\,46\mu_{X_1}\mu_{X_6} \\ & - 0.000\,52\mu_{X_2}\mu_{X_4})^2.\end{aligned}$$

Bottom deviation:

$$\mu_2 = 0.001\,41 + 0.001\,76\mu_{X_1} + 0.000\,95\mu_{X_2} + 0.003\,79\mu_{X_3} - 0.000\,38\mu_{X_5} - 0.000\,23\mu_{X_6} + 0.000\,87(\mu_{X_1}^2 + \sigma_{X_1}^2) + 0.000\,59(\mu_{X_3}^2 + \sigma_{X_3}^2) - 0.001\,21(\mu_{X_7}^2 + \sigma_{X_7}^2) + 0.001\,32\mu_{X_1}\mu_{X_2} + 0.000\,41\mu_{X_1}\mu_{X_3} + 0.000\,53\mu_{X_2}\mu_{X_3} - 0.000\,36\mu_{X_3}\mu_{X_6}.$$

With a variance, $\sigma_{\varepsilon_2}^2 = 0.000\,001$, and partitioned error variance:

$$\sigma_{P2}^2 = 0.254\sigma_{\varepsilon_2}^2 + (0.0005 - 0.000\,26\mu_{X_1} - 0.000\,23\mu_{X_3} + 0.000\,29\mu_{X_4} + 0.000\,28\mu_{X_7} - 0.000\,28\mu_{X_1}\mu_{X_7} - 0.000\,29\mu_{X_3}\mu_{X_7} + 0.000\,17\mu_{X_4}\mu_{X_6} + 0.0003\mu_{X_4}\mu_{X_7})^2.$$

Left deviation:

$$\mu_3 = 0.011\,93 - 0.002\,07\mu_{X_1} + 0.001\,79\mu_{X_2} + 0.004\,19\mu_{X_4} - 0.001\,59\mu_{X_7} - 0.003\,78(\mu_{X_4}^2 + \sigma_{X_4}^2) - 0.002\,07\mu_{X_1}\mu_{X_2} - 0.002\,03\mu_{X_1}\mu_{X_7} + 0.004\,17\mu_{X_4}\mu_{X_7}.$$

With a variance, $\sigma_{\varepsilon_3}^2 = 0.000\,036$, and partitioned error variance:

$$\sigma_{P3}^2 = 0.263\sigma_{\varepsilon_3}^2 + (0.001\,22 + 0.000\,68\mu_{X_2} - 0.001\,15\mu_{X_3} + 0.000\,63\mu_{X_5} + 0.002\,54\mu_{X_7} + 0.002\,87(\mu_{X_4}^2 + \sigma_{X_4}^2) + 0.001\,58(\mu_{X_6}^2 + \sigma_{X_6}^2) - 0.000\,97\mu_{X_3}\mu_{X_4} - 0.001\,98\mu_{X_3}\mu_{X_7})^2.$$

Right deviation:

$$\mu_4 = 0.009\,01 + 0.000\,73\mu_{X_2} + 0.001\,17\mu_{X_3} - 0.000\,14\mu_{X_4} + 0.002\,44\mu_{X_5} + 0.002\,14\mu_{X_6} + 0.000\,36\mu_{X_7} - 0.000\,64(\mu_{X_1}^2 + \sigma_{X_1}^2) - 0.000\,66(\mu_{X_4}^2 + \sigma_{X_4}^2) + 0.000\,74(\mu_{X_6}^2 + \sigma_{X_6}^2) + 0.000\,44\mu_{X_2}\mu_{X_3} - 0.000\,34\mu_{X_2}\mu_{X_6} - 0.000\,41\mu_{X_3}\mu_{X_5} - 0.000\,53\mu_{X_4}\mu_{X_5} + 0.001\,53\mu_{X_5}\mu_{X_6}.$$

With a variance, $\sigma_{\varepsilon_4}^2 = 0.000\,001$, and partitioned error variance:

$$\sigma_{P4}^2 = 0.257\sigma_{\varepsilon_4}^2 + (0.000\,64 + 0.0001\mu_{X_3} + 0.000\,07\mu_{X_5} - 0.000\,32(\mu_{X_1}^2 + \sigma_{X_1}^2) - 0.000\,27(\mu_{X_2}^2 + \sigma_{X_2}^2) + 0.000\,28(\mu_{X_3}^2 + \sigma_{X_3}^2) - 0.0004(\mu_{X_4}^2 + \sigma_{X_4}^2) + 0.000\,41(\mu_{X_5}^2 + \sigma_{X_5}^2) - 0.000\,08\mu_{X_2}\mu_{X_5} + 0.000\,11\mu_{X_3}\mu_{X_5} + 0.0001\mu_{X_3}\mu_{X_6} + 0.000\,13\mu_{X_5}\mu_{X_6})^2.$$

Quality of print:

$$\mu_5 = 9.669 - 0.25\mu_{X_2} + 1.222\mu_{X_3} - 0.296\mu_{X_4} + 1.528\mu_{X_5} + 0.287\mu_{X_6} + 0.343\mu_{X_7} - 1.888(\mu_{X_3}^2 + \sigma_{X_3}^2) - 2.638(\mu_{X_5}^2 + \sigma_{X_5}^2) + 1.362(\mu_{X_6}^2 + \sigma_{X_6}^2) - 0.638(\mu_{X_7}^2 + \sigma_{X_7}^2) + 0.802\mu_{X_2}\mu_{X_5} + 1.156\mu_{X_3}\mu_{X_5} - 0.427\mu_{X_5}\mu_{X_6} + 0.448\mu_{X_5}\mu_{X_7}.$$

With a variance, $\sigma_{\varepsilon_5}^2 = 1.26$, and partitioned error variance:

$$\sigma_{P5}^2 = 0.209\sigma_{\varepsilon_5}^2 + (0.8199 + 0.1831\mu_{X_1} - 0.1106\mu_{X_2} + 0.1725\mu_{X_3} - 0.2254\mu_{X_5} - 0.4112(\mu_{X_1}^2 + \sigma_{X_1}^2) - 0.4274(\mu_{X_2}^2 + \sigma_{X_2}^2) + 0.7101(\mu_{X_3}^2 + \sigma_{X_3}^2) + 1.1545(\mu_{X_5}^2 + \sigma_{X_5}^2) - 0.5307(\mu_{X_6}^2 + \sigma_{X_6}^2) - 0.6603(\mu_{X_7}^2 + \sigma_{X_7}^2) - 0.1319\mu_{X_1}\mu_{X_5} + 0.3351\mu_{X_2}\mu_{X_5} - 0.1385\mu_{X_3}\mu_{X_5})^2.$$

The screening of printed circuit board panels problem was formulated using (4) for maximizing *MCpk* and (7) for maximizing *MCpm*. To illustrate the flexibility of (4) we investigate several interesting aspects of the screening problem. In particular, problems were formulated for: (i) the constant variance case; (ii) the robust design case; (iii) the propagation of error case; (iv) a combination of robust design and propagation of error cases; (v) maximize the minimum capability case; (vi) increased cost for print quality case; and (vii) minimum capability case. To facilitate discussion, we shall refer to these cases as C1, C2, C3, C4, C5, C6, and C7, respectively. These formulations were entered into EXCEL and "SOLVER" was used to obtain solutions, using the experimental design points as initial values. For each case, Table 1 contains a comparative listing of the resulting maximum *MCpk* and *MCpm* values, along with the corresponding individual *Cpk*'s and *Cpm*'s leading to these maximum values. To further facilitate our consideration of each case, the results in Table 1 are graphically summarized in Fig. 5(a and b). The results for the *MCpk* criterion are summarized in Fig. 5a. The individual performance measure *Cpk*'s are depicted in Fig. 5b, respectively, for top deviation, bottom deviation, left deviation, right deviation, and print quality. The graphs in Fig. 5(a and b) have been constructed such that the cases are sorted according to descending values of *MCpk* (Fig. 5a). A discussion of the results for each case follows:

C1: constant variance

For the constant variance case, the maximum *MCpk* is 2.004 (this solution would also be optimal for the desirability index). When maximizing *MCpm* the maximum value is 1.295 (the *MCpk* for this solution is 1.888).

As shown in Fig. 5a, this case achieved the third smallest *MCpk*. The smallest *Cpk* is 0.833 for the left deviation, and the *Cpk* for print quality is 1.445. Indeed for all of the cases (C1 through C7), the *Cpk* for the left deviation is always the smallest relative to the other

Table 1. The maximum (*MCpk*, *MCpm*) and corresponding (*Cpk*'s, *Cpm*'s) for each case considered in the screening of printed circuit boards study

Case	*Cpk*					*MCpk*	*Cpm*					*MCpm*
	Top	*Bottom*	*Left*	*Right*	*Print*		*Top*	*Bottom*	*Left*	*Right*	*Print*	
Case 1: constant variance	1.959	5.000	0.833	2.743	1.445	2.004	2.065	4.988	0.833	0.964	0.439	1.295
Case 2: robust design	3.371	7.552	0.781	5.313	3.166	3.198	2.352	9.444	0.852	0.766	0.961	1.694
Case 3: propagation of error	1.770	3.773	0.561	2.053	0.889	1.469	1.876	3.619	0.603	0.893	0.282	1.007
Case 4: robust design and propagation of error	3.047	4.732	0.527	3.508	0.791	1.840	2.421	4.182	0.633	0.888	0.352	1.150
Case 5: maxmin	2.658	5.142	1.311	3.014	1.311	2.344						
Case 6: cost	3.414	7.608	0.780	4.959	3.282	3.189						
Case 7: $Cpk \geq 1$	2.605	8.628	1.000	4.688	2.291	2.996						

deviation measures. Essentially this occurs since the variance for this performance measure ($\sigma^2_{t_3} = 0.000\,036$) is more than seven times that of either the top ($\sigma^2_{t_1} = 0.000\,005$), bottom ($\sigma^2_{t_2} = 0.000\,001$), or right ($\sigma^2_{t_4} = 0.000\,001$) deviations.

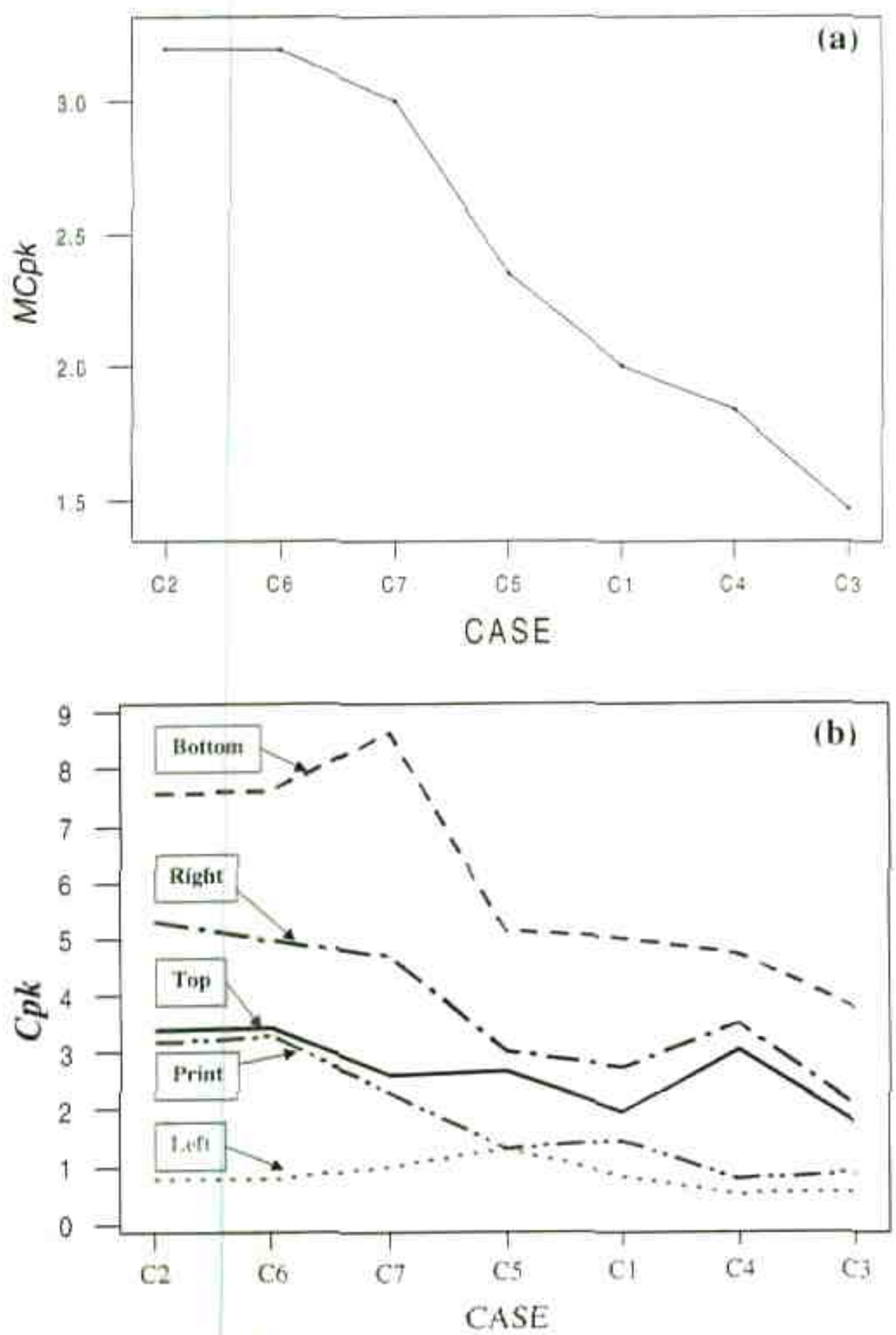


Fig. 5. (a) The *MCpk* and (b) the *Cpk* values for each case considered in the screening of the printed circuit board study.

C2: robust design

In this case, we investigate the overall level of improvement in process capability when design factor effects on performance variance are accounted for. For the robust design case, the maximum *MCpk* improved substantially to 3.198. This optimal solution was arrived at seven times out of a possible 31 initial starting points for the "SOLVER" algorithm (Starting points: one center point + 16 factorial points + 14 face-center points). As shown in Fig. 5a, the use of robust design achieved the greatest *MCpk*.

Some of the performance measures are substantially affected by robust design. For example, as shown in Fig. 5b, the *Cpk* for print quality has improved substantially from 1.445 to 3.166. However, the *Cpk* for the left deviation was reduced by robust design, from 0.833 to 0.781. Further, when robust design effects are accounted for, all of the remaining *Cpk* measures improved relative to the constant variance case.

C3: propagation of error

In this case we examine the impact of transmitted variation on overall system capability. This case highlights those performance measures that are sensitive to propagation of error. For this case, the maximum *MCpk* is reduced to 1.469 (from 3.198 in C2). The maximum *MCpm* is also reduced to 1.007. As shown in Fig. 5a, this case achieves the lowest capability, indicating substantial sensitivity to lack of control of the design factors. The left deviation remains the lowest in capability, with a substantial reduction in *Cpk* from 0.833 (for C1) to 0.561. In addition, the *Cpk* of print quality has been greatly reduced from 1.445 to 0.889. Essentially, as shown in Fig. 5b, the *Cpk*'s for the top, bottom, and right deviations were the lowest and the *Cpk*'s for left deviation and print quality were close to the lowest when propagation of error or lack of precise control in the design factors is present.

C4: robust design with propagation of error case

In this case, we study the impact of robust design in the presence of transmitted variation and compare the results

to those of case 3. This permits us to assess the extent of improvements in capability that can be attained when robust design is utilized.

For this case, the introduction of robust design improved *MCpk* from 1.469 to 1.840. Although the introduction of robust design has reduced the influence of error propagation on process capability, this case achieves the second lowest *MCpk* (Fig. 5a). As shown in Fig. 5b, the *Cpk*'s for top, bottom, and right deviation did improve. However, the *Cpk*'s for left deviation and print quality actually decreased. This further emphasizes the importance of precise control of the design factors.

C5: standardized performance measure (robust design)

In this case, we build on Case 2 (robust design) by maximizing the minimum *Cpk*, which is the standardized performance measure of Barton and Tsui (1991). Equation (10) is used to maximize the minimum *Cpk*. As shown in Fig. 5b, this case achieves the largest *Cpk* for the left deviation. However, as shown in Fig. 5(a and b), this was accomplished by a substantial reduction in the *Cpk*'s of print quality (from 3.166 to 1.311), top deviation (from 3.371 to 2.658), and right deviation (from 7.552 to 5.412), resulting in a substantial reduction in *MCpk* (from 3.198 to 2.334). Thus, given the relatively low capability of print quality, maximizing the minimum *Cpk* does not provide a satisfactory approach for optimizing this screen printing process.

C6: adjusting nonconformance costs (robust design)

In this case, we build on case 2 for robust design by increasing the nonconformance cost of print quality relative to the deviation performance measures. We incorporate these costs using Equations (11a) and (11b) from Section 2.2. We consider the case where the cost of nonconforming print quality is twice the nonconformance costs of each of the deviation measures. Further, this case achieved the second largest *MCpk* of 3.189. The *Cpk* of print quality was improved from 3.166 to 3.282. However, this was accomplished by slightly lowering the *Cpk*'s for both the left and right deviation performance measures.

C7: lower bound on each Cpk of one (robust design)

In this case, we also build on case 2 for robust design by placing a lower bound of one on the *Cpk* of each performance measure. In case 5 we were able to achieve a lower bound on each *Cpk* of 1.311. However, this was at the expense of greatly reducing the capability of print quality from 3.166 to 1.311. Consequently, we wish to determine if it is possible to obtain at least a *Cpk* of 1.00 without drastically reducing the capability of print quality. To accomplish this we place a lower bound of one on the capability of each performance measure. This lower bound can be invoked by modifying (4d) with $C_i \geq 1$.

As shown in Table 1 and Fig. 5(a and b), the resulting capabilities are *Cpk* = (2.605, 8.628, 1.000, 4.688, 2.291) with an *MCpk* = 2.996. Thus, we can achieve a *Cpk* of one for left deviation and retain at least a *Cpk* of 2.291 on the remaining performance measures. Further, the impact on *MCpk* is small since it has been only slightly reduced from 3.198 to 2.996, which is the third largest capability achieved in this study.

5. Summary

Several issues have been addressed and several methods have been proposed for considering maximization of process capability as the criterion for assessing and selecting product and process designs. For both single and multiple performance measure designs, the *MCpk* criterion and the *MCpm* criterion have been introduced and developed. The similarity to and advantages of the *MCpk* criterion with respect to the desirability index (Derringer and Suich, 1980; Harrington, 1965) has been demonstrated. Nonlinear mathematical programming formulations have been developed for determining the design factor values that maximize *MCpk* and *MCpm*. It was shown that the formulation for maximizing *MCpk* may also be used to maximize the desirability index. Further, the standardized performance measure of Barton and Tsui (1991) was shown to be a special case of the formulation for maximizing *MCpk*.

A method for including propagation of error, or the influence of design factor variance on the variance of a performance measure, when maximizing *MCpk* and *MCpm* has also been developed. In addition to the propagation of error, a method has been proposed for identifying design factors that influence residual variance, aka robust design, and for assessing the extent of such influence via the use of a response surface design. A straightforward means for detecting alternative solutions and assessing the optimality of solutions has also been proposed and developed. Implementations of the formulations for maximizing *MCpk* and *MCpm* have been demonstrated using two examples. To demonstrate the accessibility of the proposed methods to a broad range of users, each of these examples was entered into EXCEL and "SOLVER" has been used to obtain solutions. The first example is in product design and considers a VLSI electronic device design. The second example is in process design and is an original and new example in the screening of printed circuit board panels.

Overall, it has been shown that maximizing process capability provides an effective and useful criterion for optimizing product and process designs. Building upon this research, there are a number of interesting topics for further research in this area. For instance, it would be useful to investigate alternative means for accounting for cost tradeoffs, other than those developed in Section 2.

Another potential area for additional research would be to consider multiple performance measure problems where the error terms are correlated, such as in multistage serial production systems. Additionally, the method for assessing the influence of design factors on residual variance, although effective, warrants further investigation into other potentially useful mechanisms for partitioning the residual variance into that portion explained by the design factors, and that portion left unexplained. Finally, it would be useful to investigate the use of other capability criteria as alternatives to *MCpk*. For example, using *Cpkm* to develop an *MCpkm* criterion would be interesting, since *Cpkm* accounts for off-centered processes in a manner similar to that of *Cpk*.

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Biography

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Contributed by the Design of Experiments and Robust Designs Department.