**Minimizing Manufacturing and Quality Costs in Multiresponse Optimization**
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# Minimizing Manufacturing and Quality Costs in Multiresponse Optimization

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## *Abstract*

Most industrial processes and products are evaluated by more than one quality

characteristic.  To select the best design and operating control factors it is necessary to take into

account all measures of quality simultaneously, in what is known as multiresponse optimization.

Determination of the best operational settings for products or processes is usually

accomplished through single-objective function optimization routines.  Here we propose a

new objective function for multiresponse optimization, based on Taguchi's loss function.

Our function innovates in taking into account manufacturing and quality-related costs in the

optimization, in addition to the usual criteria of minimum response variance and distance-to-

target, and maximum robustness.  We also propose a simplified procedure for modeling the

variance of responses from non-replicated factorial designs. The paper contains a case study

from the rubber industry.

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## Introduction

In many experiments it is important to evaluate the same experimental unit with respect to more than one response. Such experiments are known as multiresponse experiments and are used in the development and improvement of industrial processes and products. Responses are measured from experimental units obtained by testing different combination of levels for the control factors. Ultimately, we want to determine the best control factors settings such that responses present some desired properties; for example, small distance-to-target and variance. Determining such settings is ordinarily done using single objective-function optimization routines. Thus, a central problem in the analysis of multiresponse experiments is finding a suitable objective function that combines several responses as well as relevant information related to them.

A number of authors have suggested utility functions that may be used as objective functions in the optimization of multiresponse experiments. Approaches in the literature may be arranged in three groups: (*i*) response surface methodology approaches, including the works of Lind *et al*. [1], Myers & Carter [2], Biles [3], Myers *et al*. [4], Myers & Montgomery [5], Del Castillo [6], and Kim & Lin [7]; (*ii*) approaches based on the desirability function, including the works of Harrington [8], Derringer & Suich [9], Chang & Shivpuri [10], Del Castillo *et al*. [11], and Fogliatto *et al*. [12]; and (*iii*) approaches based on Taguchi's Robust Design theory, including the works of Khuri & Conlon [13], León *et al*. [14], Logothetis & Haigh [15], Tribus & Szonyl [16], Yum & Ko [17], Raiman & Case [18], Elsayed & Chen [19], Pignatiello [20], Ribeiro & Elsayed [21], León [22], and Vinning [23].

Typical criteria in multiresponse optimization include: (*i*) distance-to-target of response outcomes, (*ii*) response variance, (*iii*) sensitivity of the response to small variations in the

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settings of the control factors, and (*iv*) inaccuracy of prediction. All criteria are to be minimized in the optimization. Criterion (*ii*) is recommended in cases where the variance of responses can be described as a function of the experimental control factors. Criterion (*iii*) is useful when experiments are carried out in pilot plants or labs to be eventually scaled up. Criterion (*iv*) captures the variability in the responses adjusted for how well they can be predicted, given the experiment performed to estimate the parameters in the prediction model.

All optimization criteria above relate to responses and their desired properties. However, in several experimental setups a response close to target or with small variance may imply in unacceptable manufacturing costs. Such costs relate to raw-material, labor, energy, etc. Typical examples are mixture experiments, where high usage of a given ingredient may result in undesirable costs. Some utility functions, like those based on Taguchi, search for control factor settings that minimize costs due to poor quality. Our objective is to search for an optimum that results in both low quality and low manufacturing costs.

In this paper we extend the quadratic loss function presented in Ribeiro & Elsayed [21] to include minimum manufacturing costs as an optimality criterion in multiresponse optimization. We also suggest a new assessment of the quality loss (or proportionality) coefficient in the loss function. The extended quadratic loss function is explored on a case from the rubber industry.

We also suggest a simplified approach for modeling the variance of responses in non-replicated factorial experiments. Variance modeling typically requires experiments with several replications, which is often economically infeasible in practice. Approaches in the literature use either information from replicates [24], [25], residuals from response models [26], [27], or mixed response models with noise generated through simulation [28], [29]. Our approach for modeling the variance of a response uses the residuals from the regression model for the response mean

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and does not require replications of experimental treatments.  The approach suggested here may be viewed as a simplified version of the one in Box & Meyer [30], although developed independently.

Finally, we present a five-step procedure for multiresponse optimization and apply it to the case study data.  The proposed steps are:  (*i*) problem identification;  (*ii*) experiment planning and execution;  (*iii*) individual modeling of response mean and variance;  (*iv*) choice of utility function and optimization criteria;  (*v*) optimization proper.  Steps (*i*) – (*iii*) cover early stages of experimentation and relate mostly to data gathering.  Steps (*iv*) and (*v*) are directly related to multiresponse optimization.

In the case study we present here a rubber mixture is to be optimized.  Control factors to be adjusted are process parameters and some ingredients in the mixture.  Samples from experimental treatments are evaluated with respect to ten response variables.  Treatments are not replicated.  Optimization goals include: (*i*) minimization of costs due to poor quality, and (*ii*) minimization of manufacturing costs.  Criteria considered in (*i*) include response distance-to-target, variance, and sensitivity to fluctuations in the settings of the control factors.  Criteria considered in (*ii*) include raw-material and energy costs.

The rest of this paper is organized as follows.  In the next section we present the quadratic loss function extended to handle manufacturing costs;  we also give the details of the proposed calculation of the proportionality coefficient in the loss function.  The third section presents the approach for variance modeling in non-replicated experiments.  The fourth section gives the procedure steps for multiresponse optimization, which are illustrated by the case study. A Conclusion closes the paper in the last section.

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## Quadratic loss function with quality and manufacturing costs

Consider a multiresponse experiment where there are $J$ responses $(j = 1,\ldots, J)$, $I$

treatment combinations $(i = 1,\ldots, I)$, and $K$ control factors $(k = 1,\ldots, K)$. The vector $\mathbf{x} = [x_1,\ldots,$

$x_K]'$ gives the settings for the control factors; if these settings correspond to treatment $i$, then $\mathbf{x} =$

$\mathbf{x}_i$. $Y_j(\mathbf{x})$ denotes the outcome of the $j^{th}$ response at $\mathbf{x}$; when $\mathbf{x} = \mathbf{x}_i$, $Y_j(\mathbf{x}) = Y_{ji}$. Response

predictions obtained from regression models are denoted by $\hat{Y}_j(\mathbf{x})$, meaning the predicted value

of response $j$ at $\mathbf{x}$; when $\mathbf{x} = \mathbf{x}_i$, $\hat{Y}_j(\mathbf{x}) = \hat{Y}_{ji}$.

A multivariate quadratic loss function was proposed by Ribeiro & Elsayed [21] and given

by:

$$\hat{Z}(\mathbf{x}) = \sum_{j=1}^{J} w_j \left[ \left( \hat{Y}_j(\mathbf{x}) - T_j \right)^2 + \hat{\sigma}_j^2(\mathbf{x}) + \sum_{k=1}^{K} \sigma_k^2 \left( \frac{\partial \hat{Y}_j(\mathbf{x})}{\partial X_k} \right)^2 \right] \qquad (1)$$

where $\hat{Z}(\mathbf{x})$ denotes the predicted loss at $\mathbf{x}$, $w_j$ is the importance weight for response $j$, $\hat{Y}_j(\mathbf{x})$ is

the predicted value of response $j$ at $\mathbf{x}$, $T_j$ is the target for response $j$, $\hat{\sigma}_j^2(\mathbf{x})$ is the predicted

variance of response $j$ at $\mathbf{x}$, and $\sigma_k^2$ is the variance of control factor $k$, as observed in shop floor

full scale operations.

The first two terms inside the brackets in equation (1) are obtained by taking the expected

value of Taguchi's univariate loss function [31]; they account for distance-to-target and variance

of response outcomes, respectively. The last term was obtained from Oh [32] and gives the

sensitivity of responses to fluctuations in the settings of the control factors. The weight $w_j$ is

given by:

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$$w_j = \frac{RI_j}{\Delta E_j^2}$$

where $RI_j$ denotes the relative importance of response $j$ and $\Delta E_j$ represents one half of the interval formed by the specification limits for response $j$. A procedure for determining $RI_j$ is given in the case study section.

Equation (1) provides dimensionless loss values proportional to the low quality in the product or process under study. In our method, loss values must be converted to dollar values. For that purpose, a proportionality constant $p$ must be determined. As originally proposed by Taguchi, $p$ gives the dollar value corresponding to one unit of loss, and is determined using expert opinion [Taguchi *et al.* (31)]. We now suggest a simplified procedure for assessing $p$.

The key to our procedure is establishing the market value of products under extreme conditions for the set of quality characteristics (QCs). The idea is intuitive: product quality varies according to how QCs comply to given specifications. A product with a QC beyond spec limits may be rated "class B" and given a lower market price. Analogously, a product with all QCs close to target may be rated "class A" and given a premium market price. By determining two such classes of a product and their respective market values, we are able to find a realistic measure of the proportionality constant $p$.

Consider, for example, a class A product rated according to readings ($\hat{y}_{1A}, \hat{y}_{2A}, \hat{y}_{3A}$) of three QCs ($j = 1,2,3$); from appropriate substitutions in equation (1), we obtain a loss value of $\hat{Z}_A$. A class B product is characterized by readings ($\hat{y}_{1B}, \hat{y}_{2B}, \hat{y}_{3B}$) of the same QCs, with a predicted loss value of $\hat{Z}_B$. Market prices are \$A and \$B for class A and class B products, respectively. The proportionality constant $p$ is then given by:

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$$p = \frac{\Delta\$}{\Delta Z} = \frac{(\$A - \$B)}{(\hat{Z}_A - \hat{Z}_B)} . \tag{2}$$

Upon determining $p$, the predicted loss $\hat{Z}(\mathbf{x})$ at a certain control factor setting $\mathbf{x}$ may be converted to dollar values as follows:

$$\hat{C}_Q(\mathbf{x}) = p \times \hat{Z}(\mathbf{x}) \tag{3}$$

where $C_Q$ represents the monetary cost of poor quality.

We now want to incorporate manufacturing costs in our search for the best control factors settings.  Start by computing relevant manufacturing costs (e.g., raw material and energy costs) at each treatment combination.  Next, build a regression model relating control factors and manufacturing costs.  Coefficients may be found deterministically.  The general form of the model is:

$$C_M(\mathbf{x}) = \mathbf{X}\beta + \mathbf{u} \quad , \tag{4}$$

where $C_M(\mathbf{x})$ denotes manufacturing costs at $\mathbf{x}$, $\mathbf{X}$ is the $(I \times R)$ information matrix of regressors with $R$ indicating the number of regressors, including the intercept, in the model,  $\beta$ is a $(R \times 1)$ vector of regression coefficients, and $\mathbf{u}$ is a $(I \times 1)$ vector of residuals (when $\beta$ is found deterministically, $\mathbf{u} = \mathbf{0}$).  Predictions from equation (4) are denoted by $\hat{C}_M(\mathbf{x})$.

An extended multivariate loss function to include manufacturing and poor quality costs is obtained by adding equations (3) and (4);  i.e.,

$$\hat{C}(\mathbf{x}) = \hat{C}_Q(\mathbf{x}) + \hat{C}_M(\mathbf{x}) \tag{5}$$

where $\hat{C}(\mathbf{x})$ denotes the overall costs incurred by running a process or manufacturing a product

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under conditions given by **x**. The objective function in equation (5) is always to be minimized.

## A simplified approach to variance modeling

An overview of our approach to variance modeling is as follows. Consider a factorial experiment with factors at $l$ levels and no replicates. That is a very common situation, since replication of treatments is usually costly and frequently not performed in practice. Suppose we determine a regression model relating outcomes of a response $j$ to levels of the control factors; only significant terms are included in the model. Using predictions from the model, we determine residuals at each treatment (i.e., difference between predicted and actual outcomes of $j$). The residuals give a measure of the variance in the experiment that could not be explained by the regression model. We want to check if residuals within a given level of a control factor differ significantly from those at other levels; when that is the case, response variance, as given by the residuals, may be considered to be a function of that factor.

There are six steps for determining regression models for the variance of a response $j$:

1.  Determine a regression model relating outcomes of response $j$ to experimental control factors.

2.  For a given control factor $k$ plot the residuals at extreme levels and inspect for noticeable differences; when that is the case, proceed to step 3; otherwise, no model for the variance of $j$ is likely to be attainable.

3.  Compute the sample variances for the residuals at each extreme level of $k$; say $s^2_{k,(-1)}$ and $s^2_{k,(+1)}$. We want to test the null hypothesis $H_o$: variances are the same at extreme levels of $k$. For that purpose, an $F$-test is performed, as follows [33]:

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$$F = \frac{s^2_{k,(s)}}{s^2_{k,(b)}},$$

for a pair of extreme levels $s$ and $b$ of $k$ ($s \neq b$), using the greater of the two variance ratios.

Whenever $F \geq F_{\alpha/2}$, variances are not the same at the two levels (use $\alpha \leq 0.05$) and should be

modeled as a function of control factor $k$. Repeat steps 2 and 3 for all $K$ control factors.

4. List control factors that have effect on the variance of response $j$ and their sample variances

   at extreme levels. Organize results in a table. An example, with $M$ control factors and

   extreme levels labeled (-1) and (+1) for low and high, respectively, is given in Table 1.

| Control factors | | | | Sample variances |
|---|---|---|---|---|
| $k=1$ | $k=2$ | ... | $k=M$ | |
| -1 | 0 | ... | 0 | $s^2_{1,(-1)}$ |
| +1 | 0 | ... | 0 | $s^2_{1,(+1)}$ |
| 0 | -1 | ... | 0 | $s^2_{2,(-1)}$ |
| 0 | +1 | ... | 0 | $s^2_{2,(+1)}$ |
| ⋮ | ⋮ | ⋱ | ⋮ | ⋮ |
| 0 | 0 | ... | -1 | $s^2_{M,(-1)}$ |
| 0 | 0 | ... | +1 | $s^2_{M,(+1)}$ |

Table 1. Sample variances at extreme control factor levels of response $j$.

5. Build regression models relating sample variances and extreme levels for the control factors;

   these are the models for the variance of response $j$.

6. Repeat steps $1 \rightarrow 5$ for all remaining responses.

Some comments are noteworthy. The procedure above does not consider higher order

terms (i.e., interactions and quadratic effects) when modeling the variance. That is precisely the

reason for only checking on the sample variances at extreme levels. The benefits of considering

higher order terms are dubious, for a number of reasons: (*i*) they are not likely to be significant

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in practice; (*ii*) estimates of higher order terms require a large number of data points, which are not usually available from factorial designs; and (*iii*) considering higher order terms would require a more complex model fitting procedure, which is not our objective.

As previously mentioned, the approach above is similar to the one in Box & Meyer [30] although developed independently. There are two main differences: (*i*) Box & Meyer provide means to incorporate interactions in the variance models; and (*ii*) determination of significant terms to be included in models is performed visually, through probability plots. Also, their approach is restricted to $2^k$ factorial designs.

## Steps for multiresponse optimization – a case study

There are five steps to optimizing multiresponse experiments; they are: (*i*) problem identification; (*ii*) experiment planning and execution; (*iii*) modeling of response mean and variance; (*iv*) choice of utility function and optimization criteria; (*v*) optimization proper. Those steps are briefly introduced in the sections that follow, and illustrated using case study data.

The case study involves a rubber product used as raw-material in the manufacturing of automobile tires. The product is manufactured at a chemical plant located in Brazil. The current formulation is based on specifications defined by clients. The experiment includes twenty-seven treatments consisting of product made with different formulations and under different processing conditions. Some of the ingredients are sulfur, synthetic rubber, and carbon black. Processing conditions relate to the total mixing time. The treatments are evaluated with respect to ten response variables; some responses are density, rheology measures (such as t10, t90 and MH), and hardness. Treatments are also evaluated regarding manufacturing costs. The goal is to find

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settings for the control factors that yield a low cost product with QCs closest to target, with

minimum variance and sensitivity. In what follows, control factors and response variables are

not identified to preserve confidentiality.

## A. Problem Identification

Problem identification comprises information gathering on relevant response variables

and control factors. In practice, listing of responses and control factors is not a complex task.

However, choosing responses that reflect QCs relevant to customers and control factors that are

most likely to affect responses is not trivial; that requires a group of individuals knowledgeable

about the product or process under study.

We propose a scheme for prioritizing responses and control factors based on Quality

Function Deployment principles [34, 35]; that is summarized in Figure 1. There are two

matrices in Figure 1. Matrix **A** relates QCs and response variables; matrix **B** relates response

variables and control factors.

There are three main inputs in matrix **A**: (*i*) a list of $P$ QCs ($p = 1,...,P$); (*ii*) importance

scores $e_p$ for items in (*i*); and (*iii*) a list of $J$ potential response variables ($j = 1,...,J$) to be

measured in the experiment. The list in (*i*) should reflect customer's expectations about the

product or process, and may be developed from joint efforts of a multifunctional team.

Importance scores in (*ii*) are based mostly on expert opinion; decision analysis methods such as

Saaty's Analytic Hierarchy Process [36] may be helpful in establishing these scores. Items in

(*iii*) correspond to measurable variables likely to describe QCs in (*i*); they are determined from

technical considerations. Elements in (*i*) and (*ii*) are written in the rows of **A**; elements in (*iii*)
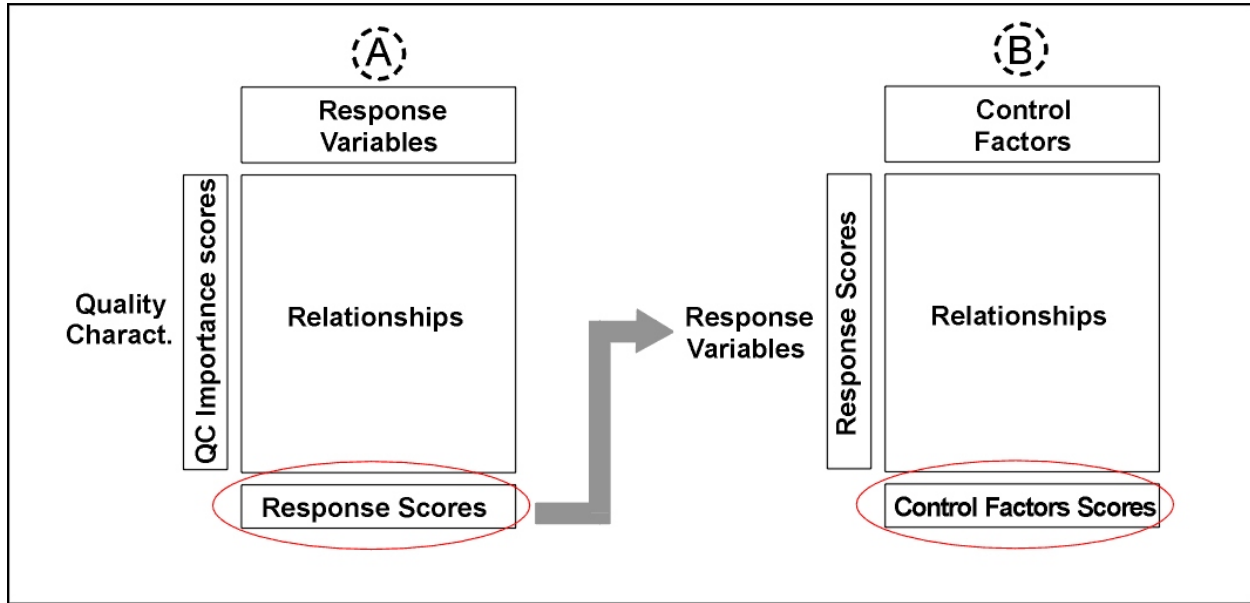
are written in the columns of **A**.

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Figure 1.　　Matrices relating quality characteristics, responses, and control factors.

Each cell $(p, j)$ in matrix **A** gives a quantitative measure $a_{pj}$ of the relationship between row element $p$ and column element $j$. Evaluations are based on expert opinion and use a continuous scale with values from 0 (no relationship) to 9 (very strong relationship). An importance ranking for response variables may be obtained from scores given by:

$$e_j = \sum_{p=1}^{P} e_p \times a_{pj} , \text{ for all } j. \tag{6}$$

Scores in equation (6) give the relative importance (RI) of responses, to be used when determining $w_j$ in equation (1).

Inputs in matrix **B** are: (*i*) a list of high scored response variables from matrix **A**; (*ii*) importance scores for the response variables in (*i*); and (*iii*) a list of potential control factors to be explored in the experiment. Items in (*iii*) are determined based on technical considerations. Relationships in **B** are determined following the same reasoning presented for matrix **A**. An importance ranking for control factors may be determined based on scores obtained from

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equation (6) and the information in matrix **B**.

The choice of responses and control factors to be considered in the experiment is based

on their importance scores, as given in matrices **A** and **B** above.  For each selected response list:

(*a*) relative importance;  (*b*) type (larger-is-best, smaller-is-best or nominal-is-best);  (*c*) current

observed value (if any);  (*d*) target value;  and (*e*) specification limits.  For each selected control

factor list: (*a*) current setting (if any);  (*b*) allowable setting interval; and (*c*) flexibility (indicates

difficulty in changing settings of a control factor).

Table 2 gives the responses selected in the case study and related information.  Relative

importance values in that table where obtained from a matrix similar to matrix **A** in Figure 1 and

normalized to a [0,5]-scale with values rounded up to the next integer.  For smaller-is-best

(larger-is-best) responses, targets correspond to threshold values from which any smaller (larger)

values are equally desirable and correspond to a zero loss situation.

Table 3 gives the control factors selected in the case study and related information.

Flexibility is measured in a [0, 10]-scale, where 10 denotes a control factor easily adjustable.

## B.  Experiment Planning and Execution

This section explores technical aspects to be considered when choosing an experimental

design.  Since the actual choice of design depends on each particular experimental setup, we

restrict ourselves to providing general guidelines.  Three main criteria direct the choice of

experimental design; they are:  (*i*) number of control factors; (*ii*) functional relationship between

responses and control factors; and (*iii*) control factor flexibility.

A large number of control factors leads to experiments where many treatment

combinations must be tested.  For example, testing 5 factors at two levels each results in 32

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treatment combinations.  When large experiments are economically infeasible, fractional designs

may be a good alternative. Thus, the number of control factors defines the degree of fractionating

to be applied in a design.  In fractional designs only a fraction of a replicate is run.  In general,

fractionating does not imply in significant loss of information; that is especially true as the

number of control factors increases ([37], [38]).  In any case, fractional designs can always be

augmented when additional information on a particular factor is desired.

| Response Variable | Relative Importance | Type | Current Value | Target Value | Spec Limits | |
|---|---|---|---|---|---|---|
| | | | | | Lower | Upper |
| $Y_1$ | 2 | Nominal | 8.0 | 8.5 | 7.93 | 9.07 |
| $Y_2$ | 2 | Nominal | 85 | 85 | 74.2 | 95.8 |
| $Y_3$ | 3 | Smaller | 220 | 210 | - | 232.7 |
| $Y_4$ | 3 | Nominal | 27 | 30 | 27.02 | 32.98 |
| $Y_5$ | 4 | Nominal | 62 | 62 | 59.49 | 64.51 |
| $Y_6$ | 4 | Nominal | 1.137 | 1.13 | 1.125 | 1.135 |
| $Y_7$ | 5 | Smaller | 80 | 65 | - | 78 |
| $Y_8$ | 4 | Larger | 1300 | 1400 | 1231.69 | - |
| $Y_9$ | 3 | Larger | 1900 | 2400 | 2328,02 | - |
| $Y_{10}$ | 4 | Larger | 500 | 530 | 496.42 | - |

Table 2.    Responses and related information in the case study.

| Control Factor | Current Setting | Allowable Setting Interval | | Flexibility |
|---|---|---|---|---|
| | | Min | Max | |
| $X_1$ | 4.0 | 3.0 | 5.0 | 10 |
| $X_2$ | 6.5 | 5.5 | 6.5 | 10 |
| $X_3$ | 19.5 | 18 | 22 | 10 |
| $X_4$ | 3.5 | 2.0 | 4.0 | 10 |
| $X_5$ | 8 | 8 | 12 | 10 |

Table 3.    Control factors and related information in the case study.

Expected functional relationships between responses and control factors determine the

number of levels at which control factors should be explored.  Linear relationships are well

explained by two-level designs;  nonlinear relationships, however, require three or more levels

for proper modeling.  An efficient strategy is to start experimentation using a two-level design

added of central points to check on nonlinear effects: in case they are significant, the design is

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expanded to contemplate three levels of each control factor; for details, see Myers &

Montgomery [5].

Control factor flexibility determines the degree of restriction on randomization.  Flexible

control factors are easily adjustable, with levels that may be completely randomized when

running the experiment.  Inflexible control factors are difficult to adjust and will be better held in

blocked or split-plot designs.  Consider the importance scores for the control factors determined

from the approach in Figure 1.  A low-scored inflexible control factor may be efficiently

explored if confounded with blocks in a blocked design.  A high-scored inflexible control factor

will be better treated in a split-plot design; for details, see Hicks [38].

In Table 4, we suggest some commonly used designs that comply with the criteria above.

In that table, CCD refer to Central Composite Designs [39].  Also, fractional CCD refer to CCDs

in which the factorial portion is partitioned.  In experiments where nonlinear effects are

expected, a number of factors greater than two may be considered large; when expecting only

linear effects, a number of factors greater than four may be considered large.  For more

information on suggested designs, see [37] and [38], among others.

| | | Number of control factors | |
|---|---|---|---|
| | | Small | Large |
| High Flexibility | Linear | $2^k$ factorial | $2^k$ fractional factorial Plackett-Burman |
| | Nonlinear | $3^k$ factorial CCD | $3^k$ fractional factorial Fractional CCD |
| Low Flexibility | Linear | Same as above. Use blocking when dealing with low-scored control factors. | |
| | Nonlinear | Use a split-plot strategy when dealing with high-scored control factors. | |

Table 4.    Recommended experimental designs according to selected criteria.

Table 5 gives the design matrix used in the rubber experiment.  Given the large number

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of factors, high control factor flexibility and possibility of nonlinear effects, a fractional CCD

was chosen. The experiment was run in three blocks. The first two blocks (eighteen

experimental runs) correspond to a $2^{5-1}$ fractional factorial with two center points added to check

on nonlinear effects, which resulted significant. According to technicians knowledgeable about

the product, only control factors $X_1$ and $X_2$ could present nonlinear effects. Runs 24 to 27

correspond to the star portion of a CCD with two factors ($\alpha = 1$); they were run in a third block

with five additional center points (runs 19 to 23). Notice that factors $X_3$, $X_4$, and $X_5$ were held at

center values in the second block. Runs were randomized within each block.

## C. Modeling of response mean and variance

We want to develop regression models for the mean and variance of responses, using an

appropriate modeling technique. There are four regression techniques available in the literature,

each related to a different set of assumptions; they are: OLS (ordinary least squares regression),

GLS (generalized least squares), MVR (multivariate regression), SURE (seemingly unrelated

equations regression). OLS and GLS model responses individually; MVR and SURE model

responses simultaneously. Choice of technique is driven by the degree of correlation among

responses and homogeneity of responses variance. A summary of the assumptions for using

each technique is given in Figure 2; these techniques are presented and compared in the context

of multiresponse optimization in Fogliatto & Albin [40].

Although not correlated among themselves, all but one of the responses in Table 2

presented non-homogeneous variances. Thus, GLS regression was used for modeling the means

of responses, with weights given by the reciprocal of the predicted variances at each treatment.

Resulting models are given in Table 6. Variance modeling followed the simplified approach

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previously presented;  resulting models are also given in Table 6.

| Run | Block | X1 | X2 | X3 | X4 | X5 | Run | Block | X1 | X2 | X3 | X4 | X5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | -1 | -1 | -1 | 1 | -1 | 15 | 2 | 1 | -1 | -1 | 1 | 1 |
| 2 | 1 | -1 | -1 | -1 | -1 | 1 | 16 | 2 | 1 | 1 | 1 | -1 | -1 |
| 3 | 1 | -1 | 1 | 1 | 1 | -1 | 17 | 2 | 1 | 1 | 1 | 1 | 1 |
| 4 | 1 | -1 | 1 | 1 | -1 | 1 | 18 | 2 | 0 | 0 | 0 | 0 | 0 |
| 5 | 1 | 1 | 1 | -1 | 1 | -1 | 19 | 3 | -1 | 0 | 0 | 0 | 0 |
| 6 | 1 | 1 | 1 | -1 | -1 | 1 | 20 | 3 | 1 | 0 | 0 | 0 | 0 |
| 7 | 1 | 1 | -1 | 1 | 1 | -1 | 21 | 3 | 0 | -1 | 0 | 0 | 0 |
| 8 | 1 | 1 | -1 | 1 | -1 | 1 | 22 | 3 | 0 | 1 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 0 | 0 | 0 | 23 | 3 | 0 | 0 | 0 | 0 | 0 |
| 10 | 2 | -1 | 1 | -1 | -1 | -1 | 24 | 3 | 0 | 0 | 0 | 0 | 0 |
| 11 | 2 | -1 | 1 | -1 | 1 | 1 | 25 | 3 | 0 | 0 | 0 | 0 | 0 |
| 12 | 2 | -1 | -1 | 1 | -1 | -1 | 26 | 3 | 0 | 0 | 0 | 0 | 0 |
| 13 | 2 | -1 | -1 | 1 | 1 | 1 | 27 | 3 | 0 | 0 | 0 | 0 | 0 |
| 14 | 2 | 1 | -1 | -1 | -1 | -1 | | | | | | | |

Table 5.    Design matrix for the rubber experiment.
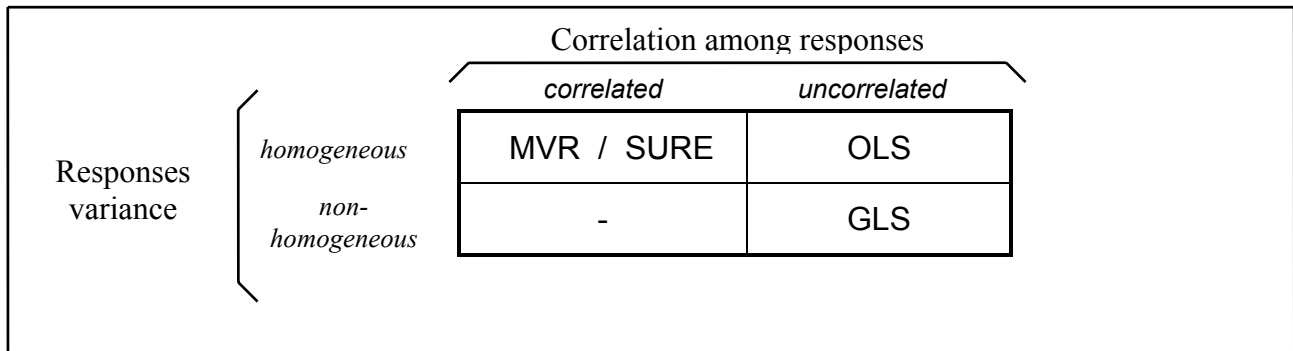


Figure 2.        Summary of assumptions for using OLS, GLS, SURE and MVR regression.

Contour plots were obtained using the models for the mean of each response and some of

the control factors.  Figure 3 presents those plots for responses $Y_1$ to $Y_4$.  It is clear from those

plots that responses are closest to their targets at different control factors settings, demanding

simultaneous optimization.

Recall that control factors correspond to process parameters and ingredients in a formula.

Such ingredients in predefined quantities are processed in a special equipment, to give a desired

rubber product.  Process parameter and measurement variations are likely to occur when

weighing and processing ingredients; these errors may be estimated to give a variability measure

for the control factors to be used in equation (1). Variability associated with the control factors

in the case study was measured from full scale in-plant operations. Estimates of $\sigma_k^2$, $k = 1,\ldots,5$

are given by:

$$\hat{\sigma}_1 = 0.16; \ \hat{\sigma}_2 = 0.06; \ \hat{\sigma}_3 = 0.05; \ \hat{\sigma}_4 = 0.12; \ \hat{\sigma}_5 = 0.2. \tag{7}$$

| 1 | Mean | $\hat{Y}_1 = 7.10 + 1.08X_1 + 0.64X_1^2 + 1.11X_2 + 0.54X_2^2 + 0.42X_4 + 0.256X_1X_2$ (0.91) |
|---|---|---|
| | Var. | $\hat{\sigma}_1 = 0.362 + 0.186X_1 + 0.118X_2$ |
| 2 | Mean | $\hat{Y}_2 = 74.62 - 2.33X_1 - 6.26X_2^2$ (0.39) |
| | Var. | $\hat{\sigma}_2 = 4.125 - 1.40X_3 + 1.58X_5$ |
| 3 | Mean | $\hat{Y}_3 = 201.19 - 4.89X_1 - 7.78X_1^2 - 3.89X_2 - 9.8X_5$ (0.79) |
| | Var. | $\hat{\sigma}_3 = 6.225 + 2.525X_1$ |
| 4 | Mean | $\hat{Y}_4 = 31.57 + 3.60X_1 + 1.43X_1^2 + 1.98X_2 + 1.58X_2^2 + 1.69X_3 + 1.10X_4 + 2.36X_5$ (0.93) |
| | Var. | $\hat{\sigma}_4 = 0.623 + 0.253X_2$ |
| 5 | Mean | $\hat{Y}_5 = 61.73 + 2.06X_1 + 2.46X_1^2 + 2.33X_2 + 0.938X_3 + 0.938X_5$ (0.82) |
| | Var. | $\hat{\sigma}_5 = 1.633 + 0.892X_1$ |
| 6 | Mean | $\hat{Y}_6 = 1.132 + 0.00678X_1 + 0.0058X_1^2 + 0.0108X_2 + 0.0063X_2^2 - 0.00281X_4$ (0.86) |
| | Var. | $\hat{\sigma}_6 = 0.00356 + 0.00202X_1 + 0.00825X_4$ |
| 7 | Mean | $\hat{Y}_7 = 74.11 - 1.17X_1 - 4.88X_4 + 1.47X_5 + 0.92X_1X_2 - 0.689X_3X_4$ (0.92) |
| | Var. | $\hat{\sigma}_7 = 0.5$ |
| 8 | Mean | $\hat{Y}_8 = 1602 + 335.56X_1 + 179.47X_1^2 + 228.67X_2 + 154.47X_2^2 + 167.75X_3 + 125.75X_5$ (0.91) |
| | Var. | $\hat{\sigma}_8 = 74.92 + 26.095X_2$ |
| 9 | Mean | $\hat{Y}_9 = 3306.4 - 76.24X_1^2 - 123.44X_4 + 41.19X_1X_2$ (0.75) |
| | Var. | $\hat{\sigma}_9 = 53.03 - 23.56X_1 + 18.52X_2$ |
| 10 | Mean | $\hat{Y}_{10} = 520.7 - 58.1X_1 - 32.6X_1^2 - 34.2X_2 - 22.6X_2^2 - 32.7X_3 - 12.1X_4 - 21.6X_5$ (0.93) |
| | Var. | $\hat{\sigma}_{10} = 13.329 - 6.566X_2 - 6.673X_3$ |

Table 6. Regression models for mean and variance of responses ($R^2$ given in parentheses).

## D. Choice of Utility Function and Optimization Criteria

A list of utility functions and criteria that may be considered when optimizing

multiresponse experiments was given in the Introduction. Those utility functions consider

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different criteria and therefore may lead to different optima.  Choice of utility function is

essentially determined by the criteria to be considered in the optimization.  Upon suitable

adaptation, utility functions in the Introduction may handle any optimization criterion.
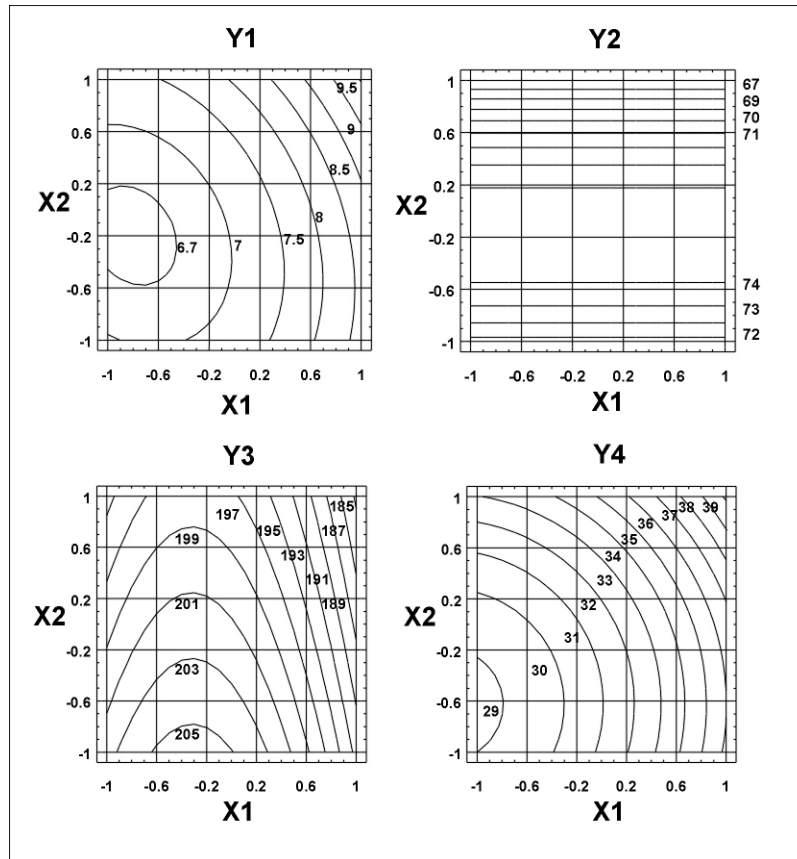


Figure 3.        Contour plots for responses $Y_1$ to $Y_4$.

The utility function used to optimize the rubber experiment is the one suggested in

equation (5), comprised of two terms:  (*i*) a measure of monetary loss due to poor product

quality, given in equation (3); and (*ii*) a measure of manufacturing costs given in equation (4).

The proportionality constant *p* to be used in equation (3), determined using our suggested

procedure, is given by:

$$p = \frac{\Delta\$}{\Delta Z} = \frac{(1.42 - 1.07)}{(179 - 64)} = 0.003043 \;.$$

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Also, a model relating manufacturing costs and control factors, as proposed in equation (4), was determined to be:

$$C_M(\mathbf{x}) = 1.42 + 0.0117X_1 - 0.0156X_2 + 0.00875X_4 + 0.00375X_5.$$

In the case, manufacturing costs included raw-material and energy costs.

We now have all the information needed to perform the optimization using equation (5) as an objective function.

### E. Optimization Proper

Upon determination of a suitable objective function (i.e., a utility function combining responses and relevant information related to them), nonlinear programming routines may be used to determine settings for the control factors where responses best satisfy the optimization criteria. Microsoft Excel [41] and Lindo's What's Best [42] are examples of computer packages able to perform nonlinear optimization.

Search for optimality in the rubber experiment was constrained by the design region (bounded by the upper and lower levels of the control factors); to conduct the search we used the Generalized Reduced Gradient (GRG) algorithm (Lasdon et al. [43]), a gradient-based optimization method available from Microsoft Excel [41] *Solver* option. Settings for the control factors that simultaneously optimize all responses and criteria are given in Table 7; they yield an expected cost of US$ 1.598. Settings in Table 7 yield predicted outcomes for the responses in Table 8; notice that predicted outcomes at optimality are fairly close to target values.

To allow for a sensitivity analysis on optimization results, a regression model relating predicted costs at experimental treatments [as given by equation (5)] and control factors was

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determined.  Figure 4(a) gives a contour plot obtained using that model and control factors $X_1$

and $X_2$;  similar plots were obtained varying levels of different pairs of control factors.  Figure

4(b) shows the effects on cost of varying the levels of control factor $X_1$ (all remaining control

factors are set at their optimal levels);  Figure 4(c) gives similar results for control factor $X_2$.

| Control Factors | Coded Levels | Control Factors | Coded Levels |
|---|---|---|---|
| $X_1$ | -0.645 | $X_4$ | 1.00 |
| $X_2$ | 0.475 | $X_5$ | -1.00 |
| $X_3$ | 0.955 | | |

Table 7.    Optimum settings for the control factors.

| Response | Specifications | Target Values | Predicted Value at Optimality |
|---|---|---|---|
| $Y_1$ | 7.5 − 9.5 | 8.5 | 7.66 |
| $Y_2$ | 75 − 95 | 85 | 72.10 |
| $Y_3$ | ≤ 230 | 210 | 209.14 |
| $Y_4$ | 27 − 33 | 30 | 31.49 |
| $Y_5$ | 60 − 64 | 62 | 62.49 |
| $Y_6$ | 1.125 − 1.130 | 1.13 | 1.134 |
| $Y_7$ | ≤ 85 | 65 | 67.57 |
| $Y_8$ | ≥ 900 | 1400 | 1638 |
| $Y_9$ | ≥ 2300 | 2400 | 3139 |
| $Y_{10}$ | ≥ 500 | 530 | 501.5 |

Table 8.    Predicted values of responses at optimality.

Inspecting contour plots and graphs like those in Figure 4 it was possible to determine a

subset of the design region where control factors settings yield costs inferior to US$ 1.70, which

was considered acceptable by the manufacturer.  Such subset is presented in Table 9, and is

recommended for manufacturing the rubber product.  Notice that control factor $X_1$ presents the

smallest interval of admissible settings; $X_1$ corresponds to the most critical ingredient in the

formulation.

## Conclusion

This paper extends the quadratic loss function proposed in Ribeiro & Elsayed [21] to

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include manufacturing costs as an optimality criterion in multiresponse optimization.  We also suggest a simplified approach for modeling the variance of responses in non-replicated factorial experiments.  Our approach uses residuals from the regression model for the response mean and may be viewed as a simplified version of the one in Box & Meyer [30].  Finally, we propose a five-step procedure for multiresponse optimization based on the following steps:  (*i*) problem identification;  (*ii*) experiment planning and execution;  (*iii*) individual modeling of response mean and variance;  (*iv*) choice of utility function and optimization criteria;  (*v*) optimization proper.

| Control Factors | Coded Levels | | |
|:---:|:---:|:---:|:---:|
| $X_1$ | -0.80 | ⇔ | -0.40 |
| $X_2$ | +0.20 | ⇔ | +0.90 |
| $X_3$ | +0.60 | ⇔ | +1.00 |
| $X_4$ | -0.40 | ⇔ | +1.00 |
| $X_5$ | -1.00 | ⇔ | +1.00 |

Table 9.    Subset of the design region yielding costs inferior to US$1.70.

Developments proposed here are applied to a case study from the rubber industry.  The product to be optimized is a rubber mixture.  Five control factors are considered;  they are process parameters and some key mixture ingredients.  Samples from experimental treatments are evaluated with respect to ten response variables.  Optimization goals are minimization of costs due to poor quality, and minimization of manufacturing costs.  Criteria for optimality considered are response distance-to-target, variance, sensitivity to fluctuations in the settings of the control factors, and raw-material and energy costs.

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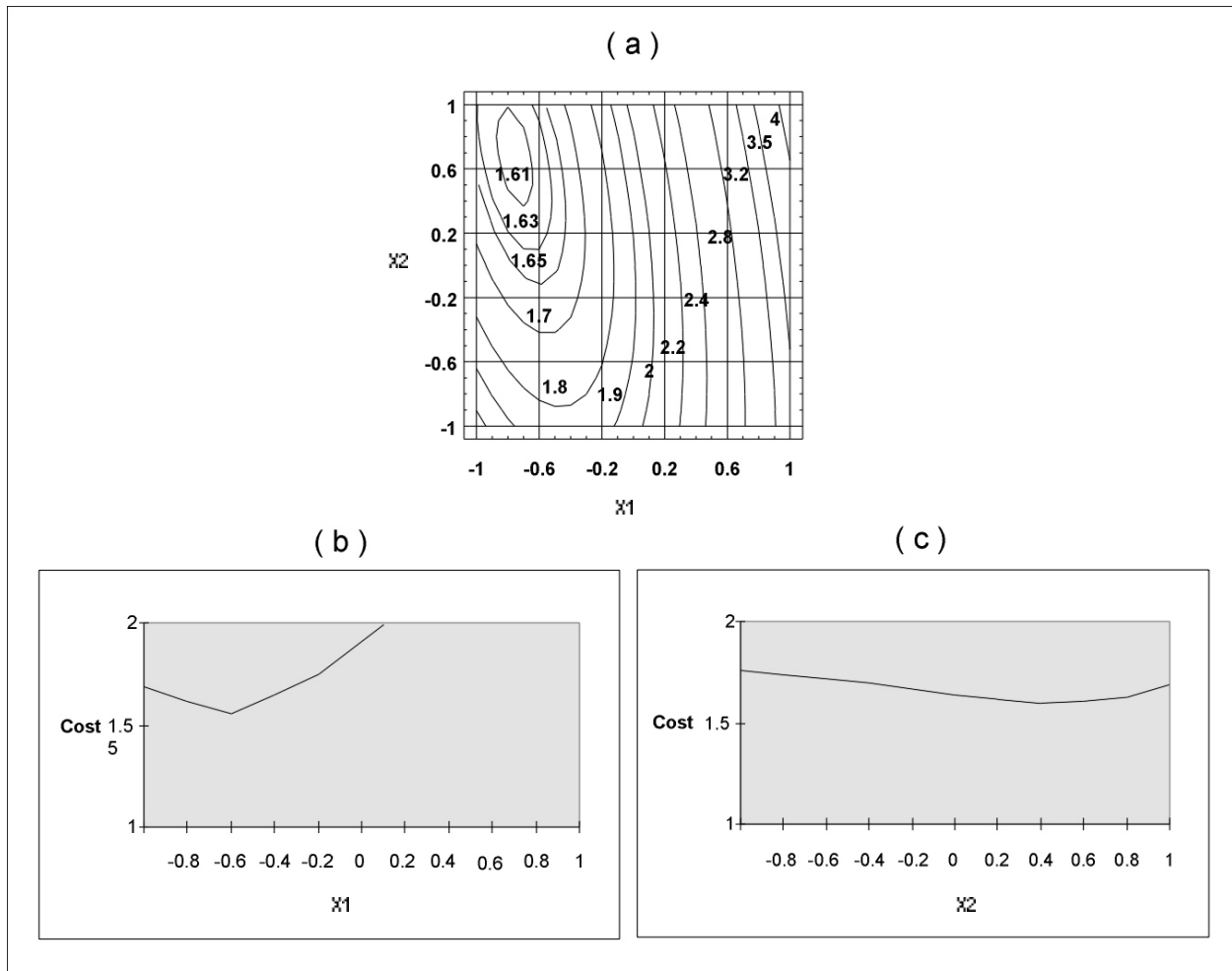
Figure 4.        Sensitivity analysis on overall predicted costs as function of control factors $X_1$ and $X_2$.

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